EXCURSE TO
THE HISTORY OF INERTIAL FORCE

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Abstract

The concept of *inertial force* is one of the most important and the most difficult in classical dynamics. It absorbed and multiplied the difficulties of both concepts of *force* and *inertia* both complex and cumbersome. However, just because of that making sense of *inertial force* may shed light on its both components. In fact, the notion of *inertial force* designates a whole cluster of meanings and addresses different subjects. In the history of physics, the term *inertial force* is used in at least five different meanings, which co-exist and usually cause confusion of novice. This topic is a good example showing that scientific knowledge is after all a *learned ignorance* (Nicholas of Cusa) but, at the same time, perhaps the only kind of knowledge associated with understanding. It is clear from the above that teaching of inertial force is not easy, but in the hands of trained teachers, the complexity and multiple meanings of "inertial force" can be a powerful tool for achieving mature understanding of classical mechanics. In fact, without inertial force and the associated with it issue of *non-inertial observer* the classical mechanics cannot suggest understanding of the most important aspect of movement – its relativity.

***Keywords**:* inertia, inertial force, centrifugal force, inertial and non-inertial observers, equivalence of inertia and gravitation, operational definition of force

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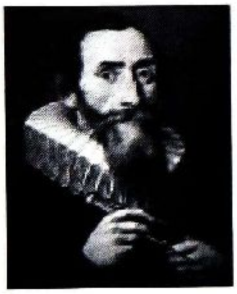

Kepler's Inertia – the force of sluggishness

Johannes Kepler (1571–1630) – a prominent German scientist – was among those few who performed the scientific revolution of the 17th century. He worked in astronomy, optics and mathematics and is considered a founder of modern physics. He is famous for his formulation of three laws of planetary motion that he elicited from the rich data accumulated by his previous employer – Tycho Brahe.

Being educated in the tradition of Aristotelian physics, Kepler shared much of the views from the Aristotelian physics, in particular, the physics of motion. However, being an original thinker and a contemporary of Galileo he did not divide the universe into two realms of different regularity – below and above the Moon (superlunary and superlunary physics) – he did not stop at mere description of circular motion of celestial bodies as natural, but continued to think about its causes, as should be done by a natural philosopher regarding the bodies in our world, being at violent (not natural) motion. Thus within this Aristotelian framework, he addressed the agent causing motion of celestial bodies:

> *If the matter of celestial bodies were not embower with inertia, something similar to weight, no force would be needed for their movement from their place; the smallest motive force would suffice to impart to them an infinite velocity. Since, however, the periods of planetary revolutions take up*

definite times, some longer and others shorter, it is clear that matter must have inertia which accounts for this differences.[1]

In this small passage an example of analysis of natural philosopher, Kepler arrived at the far-reaching conclusion regarding the nature of the universe[2]. Indeed, let us think, why the periods of planetary motions are different. Kepler relates this difference to the special feature of the celestial matter – inertia that discriminates between the planets in their ability to move under the action of force. He also points that this virtue is related to weight, similar to it. This meant for him only one thing: the heavier the body is the greater inertia it posses. Kepler argues: inertia explains the finite velocities of the planets.

We may pay attention to the fact that the inertia, as we use it with regard to Galileo's law – the law of inertia – was a very different concept. Kepler's concept addressed a different dynamical virtue of matter, its response to the exerted force. One might think that Kepler arrived to the meaning we usually ascribe to mass in classical mechanics. This, however, was not the case.

In fact, Galileo had in his mid the idea close to that of Kepler. In his famous *Dialogo*, Galileo had Sagredo (the host and chairperson of the discussion) say[3]:

> *I see in a movable body is the natural inclination and <u>tendency it has to an opposite motion</u>. . . . I said <u>internal resistance</u>, because I believe that this is what you meant, and not external resistances, which are many and accidental. (we added emphasis)*

Salviati (Galileo's representative) readily confirms and refines:

> *I wonder whether there is not in the movable body, besides a natural tendency in the opposite direction, another <u>intrinsic and natural property</u> which makes it <u>resist motion</u>* (emphasis added).

So, Galileo had a similar to Kepler's comprehending of inertia: an inherent *resistance* to motion whereas a contemporary reader would combine the *two* perceptions of inertia – resistance to *and* preservation of motion.

Kepler, who introduced the term inertia into physics, conceived of matter (of a planet) as having a faculty of impotence, an inherent 'laziness', with respect to motion, the intrinsic propensity of matter for rest. We read[4]:

> *...transporting power (potentia vectoria) of the sun and the impotence of the planet (impotentia planetae) or its material inertia strive against each other* (emphasis added)

Preventing spontaneous motion of planets, inertia of Kepler would return them to rest if the mover ceases to act: inertia represents sluggishness of the body. This similar to Galileo's idea and is very different from the Newtonian inertia. Bernard Cohen – a famous expert in Newton's legacy – made a remarkable finding when

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[1] De causis planetarum, in Keper. J., *Opera Omnia*, Vol. 6, p. 342. The quote from: Jammer, M. (1961). *Concepts of Mass in Classical and Modern Physics*. Ambridge-Massachusetts: Harvard University Press.

[2] We may put attention that Kepler created a novel problem following his breaking one of the basic presumptions of Aristotelian physics: the division of the universe into two areas with different regularities.

[3] Galiley, G. (1632/1953), p. 213.

[4] Jammer, M. op cit., p. 342.

revealed Newton's marginal remark in his private copy of the first edition of the *Principia*. Newton wrote[5]:

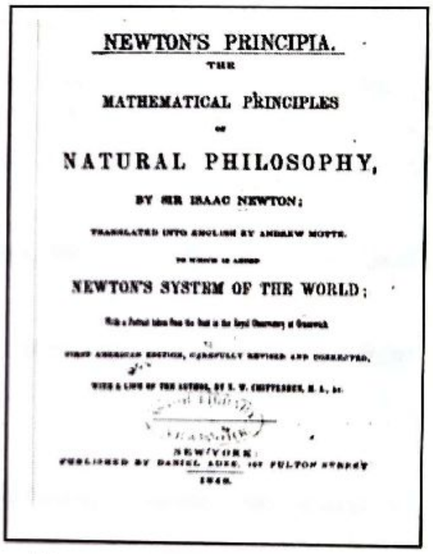
> *I do not mean Kepler's force of inertia by which bodies tend toward rest, but the* <u>*force*</u> *of remaining in the same state whether of resting or of moving.* (emphasis added)

And so we come to Newton.

Newton: inertial force as inertia of mass

Newton, Sir Isaac (1642-1727), the remarkable mathematician, physicist, and natural philosopher, one of the most brilliant scientists of all times. A Fellow of Trinity College at the University of Cambridge, he produced one of the most famous scientific books: *the Mathematical Principles of Natural Philosophy*, known as *the Principia,* published in 1687.

In his Principia, Newton defined the concept of inertial force in the very first chapter. In his third definition, he defined[6]:



NEWTON'S PRINCIPIA.

THE

MATHEMATICAL PRINCIPLES

OF

NATURAL PHILOSOPHY,

BY SIR ISAAC NEWTON;

NEWTON'S SYSTEM OF THE WORLD;

> *The vis insita, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, endeavors to persevere in its present state, whether it be of rest, or of moving uniformly forward in a right line.*

and immediately explained:

> *This force is ever proportional to the body whose force it is; and differs nothing from the inactivity of the mass, but in our manner of conceiving it. A body, from the inactivity of matter, is not without difficulty put out of its state of rest or motion. Upon which account, this vis insita, may, by a most significant name, be called* <u>*vis inertia*</u>*, or* <u>*force of inactivity*</u>*. But a body exerts this force only, when another force, impressed upon it, endeavours to change its condition; and the exercise of this force may be concedered both as resistance and impulse; it is resistance, in so far as the body, for maintaining its present state, withstands the force impressed; it is impulse, in so far as the body, by not easily giving way to the impressed force of another, endeavors to change the state of that other. Resistance is usually ascribes to bodies at rest, and impulse to those in motion; but motion and rest, as commonly conceived, are only relatively distinguished; nor are those bodies always truly at rest, which commonly are taken to be so.*

Vis insita introduced by Newton by definition was used in the first law of motion. Too often this law is considered a special case of the second one. It is true in the form

[5] Cohen, B. (1971). *Introduction to Newton's 'Principia'* University Press, Cambridge, pp. XVI, 28.

[6] Newton, I. (1687/1999). *Mathematical Principles of Natural Philosophy*. Berkeley, CA: University of California Press, p.404.

given to the first law later. Originally, it seemingly was quite different.[7] In Newton's formulation the second law refined the statement of the first law which was formulated as following:[8]

> *That every body* ***perseveres*** *in its state of resting, or of moving uniformly in a right line,* ***as far as*** *it is* ***not*** *compelled to change that state by external forces impressed upon it.* (emphasis added)

Careful reading of this text reveals that it talks about intention to preserve the state of motion when the external force is applied and not zero-force case. Traditionally, however, we keep teaching the simplified version of the first law – the special case of absence of forces, while the second law presents an exact statement of object reaction to the impressed force, claming that the inertial force (mass) inversely determines the result of impressed force action – acceleration of the body.

In his approach to describe the account for motion, Newton, unlike Galileo, sought to address the common situation of forces present whereas Galileo tried to establish the Law of Nature in its ideal form. The regular presentation of the first law in classes, therefore, goes more with the tradition coming from Galileo.

Note also the revolutionary treatment of Newton stating the full equivalence of the states of rest and motion, in contrast to Aristotelian claim ascribing a body state to be rest only while motion was considered as a *process of changing state*.[9]

In any case, vis insita described the intention of the body to preserve its state. Indicatively, in the critique of Newton by Mach at the end of the 19th century, the force of inertia was already dropped as it remained in modern formulation of mechanics[10].

Let us clarify now the relationship between the mass, the quantity of matter and inertia. We start with Newtonian Definition 1:[11]

> *The quality of matter is the measure of the same, arising from its density and bulk conjunctly.*

And in the following explanation:

> *It is this quantity that I mean here after everywhere under the name of body or mass.*

The above contains one of the most important and controversial ideas regarding motion. It was the time to clarify fundamentals in the physical account of motion. Material bodies are characterized by a variety of properties (form, weight, color etc.). Mass is not equivalent to matter, but it is the property of all material bodies. There is a reason why we say "the mass of the body" and don't say "the matter of the body". Mass is the quantified physical characteristic and the matter is not. Thus, we can say "big mass" and not "big matter". Furthermore, mass presumes certain uniformity. In

[7] Galili, I. & Tseitlin, M. (2003). 'Newton's first law: text, translations, interpretations, and physics education'. *Science and Education*, 12 (1), 45-73.

[8] Newton, I. (1687/1999). *Op. cit.* p. 416.

[9] Descartes was close to Newton's understanding but did not reach it. See the historical case of Cartesian laws of motion.

[10] Mach, E. (1893/1960). *The science of mechanics/A critical and historical account of its development.* Chicago, London: The Open Court, p. 241.

[11] Newton, I. (1687/1999). *Op. cit.* p.403.

a crowd, "in a mass", people loose individuality and behave appropriately. We use mass to emphasize the absence of differences, aiming only quantity.

Newton believed that mass indicates the quantity of the matter. But this presumes that the matter can be given us directly, as observable property of the thing. However, matter has given to us only through properties of material bodies. We have all kinds of materials (wood, metal, etc.). So, what is fundamental to any material? It cannot be the number of atoms; atoms are different. Similarly, it cannot be their volume.

We probably can assume that the quantity of matter of two identical bodies is two times more than the quantity of matter of one of them, and of three bodies - three times as much. However, how could we compare the amounts of matter in different bodies? If we cannot, then the quantity of matter cannot be the primary physical notion, describing the mass. Perhaps the quantity of matter can be secondary notion, to be expressed in terms of other concepts?

It appeared to Newton that the physics of dynamics provides such fundamental quality of material bodies for describing mass, the quality universal for all – inertia. All bodies have this feature, and so inertia describes mass of the matter. Note that gravitation is another such universal property of all material bodies, and thus the gravitation describes mass of matter too. (It was the first step to Newtonian claim of equality of gravitational and inertial masses) Since Newton believed matter (mass) is more fundamental than inertia he determined the latter in terms of the former: mass is the amount of matter.

Soon after, already in the Euler's *Mechanics* (see below) it was understood that the quantity of matter is a confusing thing, lacking operational definition. At the same time, inertia is simply measured through the force exerted on the accelerated body. Consequently, the mass as a "quantity of matter" left the rigor theoretical physics. Only mass, playing central role in the second Newton's law, the quantity of motion, and the law of gravitation, remained.

Students might feel that the quantity of the matter is a clear notion. Therefore, in introductory education, one may preserve Newtonian definition of mass as a quantity of matter. Later, in the course of study, one will discover that is more justified physically to think of mass in terms of measure of inertia, and not the quantity of the matter. As to the quantity of the matter, one may consider it as proportional to inertia, regardless any specific features of the body, or material.

Furthermore, this understanding *mass-inertia* (inertial mass) is ready for the refinement within the theory of relativity, being closer to the fundamental relation *mass-energy*. In fact, we may talk about the inertial mass of a photon, but cannot talk about its quantity of matter, since his rest-mass is strictly zero.

So, inertia is the primary property of matter and the law of inertia presents fundamental ontological claim. If for Descartes bodies were conceived by their impermeable boundary (geometry), than for us bodies boundaries are secondary after their inertia. In fact, Newton himself mentioned that the fact that masses of bodies were measured by their weights made the quantity of matter inferior to the gravitational mass. This is what he writes in the explanation to definition 1:

> *It is this quantity [the quantity of matter] that I mean hereafter everywhere under the name of body mass. And the same is known by the weight of each body; for it is proportional to the weight, as I have found by*

experiments on pendulums, very accurately made, which shall be shown hereafter.

In fact, Newton could only show that the proportionality of weight to inertial mass, from the period of pendulum oscillations, not to the quantity of matter. The proportionality to the latter was only inferred.

The definition of inertia by Newton initially included "the force of inertia". It was in some tension with definition IV which defined the external force by its effect lacking in the case of inertia. The magnitude of the internal force (vis insita), or the force of inertia, was proportional to the weight of the body (or quantity of matter). This force reflected the effort required to take the body out of the state of rest or motion.

Euler: inertia is not a force

Leonhard Euler (1707–1783) was a Swiss mathematician who greatly contributed to a wide range of topics in mathematics and physics. A young researcher he comprised his treatise *Mechanics*, the first textbook on classical mechanics, which influenced the subsequent development of mathematics, physics and philosophy. In it, he continued to use the term "inertial force" but already in the preface, he wrote:

> … a state of conservation is an essential property of all bodies, and all bodies, in as much as they are such, have the strength or facility to remain permanently in their state, which is called nothing other than the *force of inertia*. Indeed, calling the inertial effect a force for the source of this conservation is less than suitable, since *it is not a force of the same kind as the other forces* thus properly discussed, such as the force of gravity, nor can it be compared with these; into which error many are accustomed to fall, especially those involved in metaphysics, from the ambiguity of their deceptive discussions.[12]

MECHANICA
SIVE
MOTVS
SCIENTIA
ANALYTICE
EXPOSITA
AVCTORE
LEONHARDO EVLERO
ACADEMIAE IMPER. SCIENTIARVM MEMBRO ET MATHESEOS SVBLIMIORIS PROFESSORE
TOMVS I.
INSTAR SVPPLEMENTI AD COMMENTAR. ACAD. SCIENT. IMPER.
PETROPOLI
EX TYPOGRAPHIA ACADEMIAE SCIENTIARVM.
A. 1736.

Euler's definition of inertial force defines it as a faculty of a body:[13]

The force of inertia [vis inertiae] in all bodies is that in faculty of the body to maintain its state of rest or of continuing in its present state of motion in a straight line.

This definition coincides with the modern definition in physics textbooks today. One may also pay attention to the fact that the law of inertia was stated first,[14] and only after that, inertia was defined.

Since Euler wrote his *Mechanics* as a textbook, he explained things more than Newton did. In Proposition 17 Euler tried to demonstrate the important feature, which was stated by Newton as obvious:[15]

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[12] Euler, L. (1736/2008). *Mechanics or the science of motion analytically demonstrated.* Vol. 1, Preface, p. 3. Online: http://www.17centurymaths.com/contents/euler/mechvol1/preface.pdf

[13] Euler, L. (1736/2008). *Op cit.* Vol. 1, Ch. 1, Definition 9, page 18.

[14] Euler, L. (1736/2008). *Op cit.* Vol. 1, Ch. 1, Scholium 1, page 17.

The force of inertia of any body is proportional to the quantity of matter, upon which it depends.

Euler argued for this as following[16]:

> *That [the force of inertia] therefore is to be estimated from the strength or the force applied to the body, with the aid of which it is to be disturbed from its state. Truly different bodies equally [disturbed] in their state are disturbed by forces, which are as the quantities of material contained in these. Therefore, the forces of inertia of these are [also] proportional to these forces.*

In other words, we know by the second Newton's law (in the form F=ma) that if two bodies, of masses m_1 and m_2, are accelerated to the same extent ($a_1=a_2$), then the masses ("the quantities of matter") are in ratio as the forces:

$$\frac{F_1}{F_2} = \frac{m_1}{m_2}$$

In this fact, Euler saw the evidence for the forces of inertia of the bodies to be in the same ratio, the ratio of their quantities of matter: different forces of inertia, which impeded the change of motion so that they produced the same result. We demonstrated this result by using the form of the second law of motion in the form F=ma. This was not, however, the form suggested by Newton. As noted by Jammer,[17] the considered treatment of Euler was of particular importance, since starting from it the famous formula F=ma ("force equals mass multiplied by acceleration"), used by all today, eventually replaced the original Newtonian form of this central law of mechanics, apparently different:

$$\Delta(mv) = F\Delta t$$

We may say that since then the new form of the Newton's law serves us as an accurate definition of mass, as followed by the modern textbooks of physics:

$$m = \frac{F}{a}$$

In his later treatise, Euler returned to the criticism of the notion "the inertial force" we mentioned in the beginning and wrote:[18]

> ... Also now and then it is called the *force of inertia*, because the *force* is a little resistant to the change of the state; but if the force is defined by some cause, which is changing the state of the body, here it is not in the least acceptable with this meaning; certainly the reason for this strongly disagrees with that by which henceforth we show a force to be acting. Whereby, lest any confusion should arise, we omit the name force and this property of the body and we will call it by the simpler name of ***inertia***.

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[15] Euler, L. (1736/2008) op cit. Vol.1, Ch. 2, p. 59. http://www.17centurymaths.com/contents/euler/mechvol1/ch.2.pdf

[16] *Ibid.*

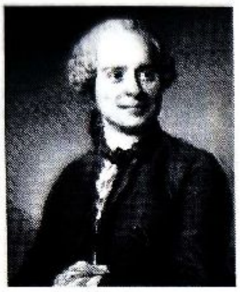
[17] Jammer, M. (1961). *Op cit.* p.88

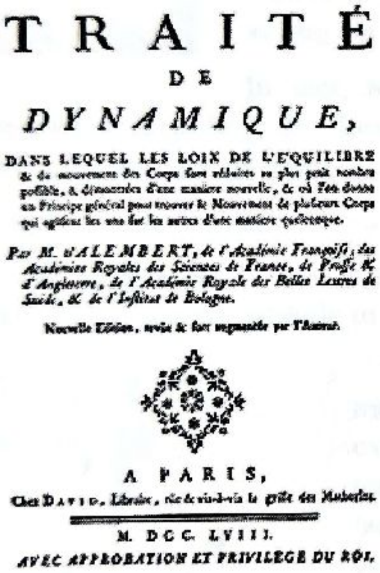
[18] Euler, L. (1765/2009). *Theory of the motion of solid or rigid bodies*. Ch. 2, p.60. http://www.17centurymaths.com/contents/mechanica3.html

In fact, Euler saw the force to be of active origin that exerted on the body causing it to change its state, while inertia was due to the innate inert matter. Therefore, he inferred, the termed "the force of inertia" is contradictive.

D'Alembert: inertial force as a fictitious force in Newton's second law

The great French scientist and philosopher Jean le Rond d'Alembert (1717-1783) apparently was the first who introduced a very different approach to the force of inertia.[19] D'Alembert started with the adoption of the idea that force of inertia as the property of bodies to remain in their state of rest or motion. However, unlike Euler, for whom this force was fundamental, d'Alembert considered such approach obscure and wanted to define all forces by their result – the change of motion.





TRAITÉ

DE

DYNAMIQUE,

DANS LEQUEL LES LOIX DE L'EQUILIBRE

Par M. D'ALEMBERT,

A PARIS,

M. DCC. LVIII.

AVEC APPROBATION ET PRIVILEGE DU ROI.

For this purpose, Newton's second law in the form F = ma is the best form. And so one may see this law as a definition of the force – "effective (or moving) force"[20].

Futhermore, within his rationalist intention to construct mechanics from solid principles d'Alembert (under the influence of Archimedes treatment of lever) sought to reduce the physics of motion to the static principle of equilibrium[21]. And so was done. Simple manipulation with the second law yields:

$$\mathbf{F} - \text{ma} = 0 \qquad \boxed{\mathbf{I} = -\text{ma}} \qquad \mathbf{F} + \mathbf{I} = 0$$

Here a new force I was precisely defined – *the force of inertia.* Was it the old force of inertia? Yes, it was. But if the old force was defined qualitatively and thus obscurely ("tendency"), the new one was given a precise mathematical formula. To support this idea, one may think about force I intuitively, as that "resistance to the change of motion", which nullifies the effect of an impressed external force. Together, the external agent and internal resistance create a sort of balance, equilibrium. We have arrived to the d'Alembert principle[22]:

Any system of forces is in equilibrium if we add to the impressed forces the forces of inertia.

Lagrange mentioned that d'Alembert reduced theory of motion to Statics. This, however, is only an illusion. Dynamics is within the expression for I (exactly: in the concealed acceleration a). Many physicists (e.g. Mach) saw it and said that d'Alembert did not say anything new: no new physics can be obtained by transferring the term from one side of equation to another.

[19] D'Alembert, J. (1758). *Traité de dynamique.* Paris: David Libraire.

[20] Later this idea to define force was adopted by other physicists, supporters of positivist philosophy, such as Mach, Hertz and others.

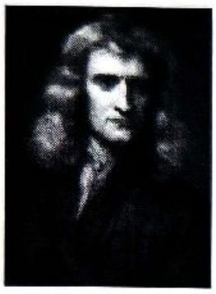
[21] We see here how strongly embedded in people the conceptual inequality between the state of rest and the state of motion. The preference of Statics, the state of rest, over Dynamics, is purely Aristotelian.

[22] This is the modern formulation of the principle as can be seen in Lanczos, C. (1949/1970). *Variational principles of mechanics.* Toronto: University of Toronto, p. 90.

Indeed, the essential difference between the forces, inertial and external was that the former was always interactive (Newton's third law) and the latter not. For the physicists of Newton's mind (absolute and unique space-time) d'Alembert force of inertia was a mere hoax, and so it got an inferior status and even the name – *fictitious force*.

New vision: Newton versus Huygens

The influence of Newton on the adopted framework of physics thought was enormous. However there was another brilliant mind – Christiam Huygens (1629-1695) a prominent Dutch physicist which worked almost in parallel with Newton and produced alternative ideas regarding the most fundamental issues of physics both in mechanics and optics. In the latter they confronted the ideas of particle (light rays) paradigm with the wave theory of light (elastic distortions in the ether medium).



Here we touch on another fundamental difference with regard to the nature of centrifugal force – manifestation of inertia in the circular motion.

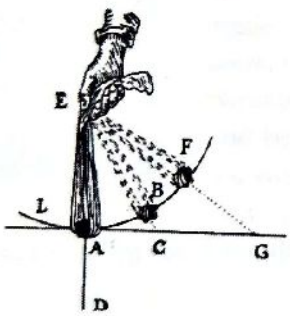
We will present the two approaches one after the other, starting with Newton.

Newton: The inertial force as the force of accelerated body acting on the constraint

In fact, in this subject, Newton followed the first thoughts of Descartes regarding the forces active in a circular motion. Although Descartes excluded the idea of inertial force.

I do not recognize any inertia or natural sluggishness in bodies…

wrote Descartes in his letter to Mersen in December 1630[23] explaining him his attitude to inertia.



However, describing the circular motion of a stone in a sling, Descartes explicitly mentioned that following the tendency of a stone to proceed in tangential direction AG (Fig.) the centrifugal effect is experienced by the sling: a radial tension striving to stretch the sling outwards, along AD, manifesting endeavor (conatus, in Latin). This was a normal consequence of the body's tendency to depart the circle along a straight tangential path.

Newton adopted this view and considering the body moving inside a solid circular frame along the sides of inscribed polygon[24]:

> *And if a body, moved with a given velocity along the sides of the polygon, is reflected from the circle at the several angular points, the force, with which at every reflection it strikes the circle, will be as its velocity: and therefore the sum of the forces, in a given time, will be as that velocity and the number of reflections conjunctly; …* <u>*This is the centrifugal force, with*</u>

[23] Descartes, R. (1976), Letter to Mersen. In *Oeuvres de Descartes*, C. Adams and P. Tannery (eds.), Paris: J. Vrin, Vol. 2, p. 466.

[24] Newton, I. (1687/1999). *Op. cit.*, Book I, Section 2, Scholium, p. 452-453.

which the body impels the circle; and to which the contrary force, wherewith the circle continually repels the body towards the centre, is equal.

Although Newton was not consistent in using the term centrifugal force, we may mention that at least the mentioned here his of the term use is in accordance with Newton's third law. Much later, Hertz,[25] expanded on this meaning of the centrifugal force: [26]

> *We swing a stone attached to a string in a circle; we thereby consciously exert a force on the stone; this force constantly deviates the stone from a straight path, and if we alter this force, the mass of the stone or the length of the string, we discover that indeed the motion of the stone occurs at all times in agreement with Newton's second law. Now, the third law demands a force opposing that which is exerted by our hand on the stone. If we ask for this force, we obtain the answer familiar to everybody, that the stone reacts on the hand by virtue of the centrifugal force, and that this centrifugal force is indeed equal and opposite to the force exerted by us on the stone. ...*

In this meaning the centrifugal force with its pair fits the Newtonian framework of forces. In summary, Newton's mechanics, the one that became classical, suggested the framework of mechanical description of the reality in terms of interactive forces only. No inertial forces were required. The place for inertia was in inertial mass only. This description was later identified with so called *inertial observers* (or *inertial frame of reference*). This, of course, could happen only when other options emerged.

Huygens: The inertial force – the force for the non-inertial observer

Huygens view was highly skilful and original. He was among few who supported the wave theory of light while the great success of Newtonian ray theory in his approach to optics. Huygens also made important contribution to mechanics, proving first the conservation of *vis viva* in elastic collision of hard bodies. Before Newton, in 1659, that is, before the introduction of inertial mass, and before the great invention of Newtonian picture of gravitation, his original mind produced the concept of *centrifugal force*[27] and determined its variables. Today we express this force in modern way, by using the formula:

$$F = \frac{mv^2}{r}$$

Huygens could only describe the dependence of what he considered to be centrifugal force on different parameters: weight, velocity and radius of rotation, often through comparison between two bodies in a similar situation.

[25] Heinrich Rudolf Hertz (1857 – 1894) was a prominent German physicist. In his "Principles of Mechanics Presented in a New Form" (1899) Hertz tried to create a new version of mechanics, reconsidering the concept of force.

[26] Hertz, H., *Mechanics* /Collected Works, Vol. III, p. 6

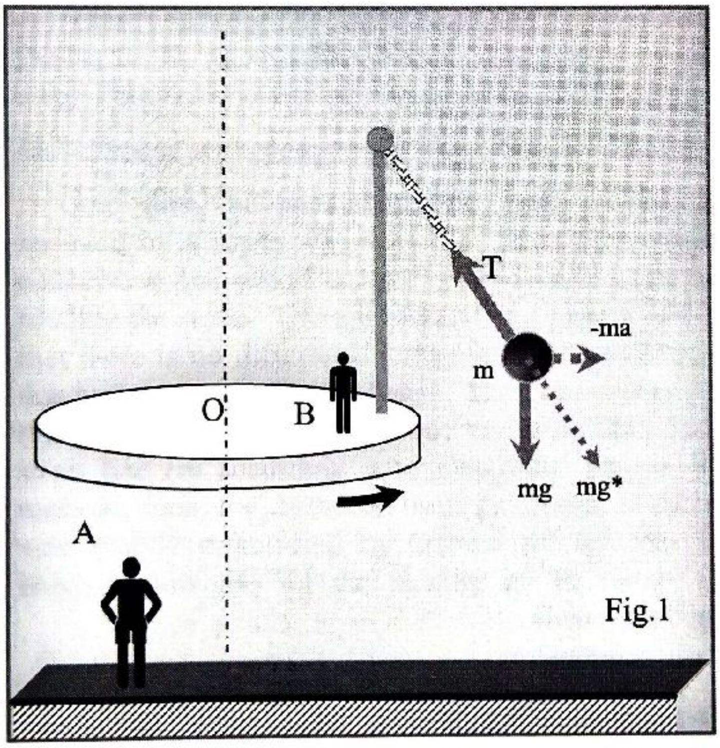
[27] Huygens, Ch. (1659/1703). On centrifugal force. From *De vi Centrifuga*, in *Oeuvres Complètes*, Vol. XVI, pp. 255-301, Translated by M.S. Mahoney.
Online: http://www.princeton.edu/~hos/mike/texts/huygens/centriforce/huyforce.htm

However, one should not forget that Huygens never wrote this formula. Moreover, before Newton people thought only in terms of weight, which was not distinguished neither from inertial mass nor from the gravitational force, both inventions of Newton. Also the concept of force itself was ambiguous and not well defined concept, waiting for Newton's touch. And despite all these, Huygens was the first who tried to describe physical situation from the point of view of the observer in a rotating system. For that he used a thought experiment suggesting:[28]

> *Let us imagine some very large wheel, such that it easily carries along with it a man standing on it near the circumference but so attached that he cannot be thrown off; let him hold in his hand a string with a lead shot attached to the other end of the string.*

He made a statement regarding the tension in a string caused by the rotation and promised to clarify this dynamic situation: [29]

> *Let us imagine some very large wheel, such that it easily carries along with it a man standing on it near the circumference but so attached that he cannot be thrown off; let him hold in his hand a string with a lead shot attached to the other end of the string.*



To facilitate understanding of Huygens we should first present the modern account for this case. In our view, we distinguish between the description of situation made by the observer A, outside the wheel (Figure 1) and that by the observer B, on the wheel, whom considered Huygens.

Observer A, considered later by Newton, mentions the tension of the rope T and the gravitational force mg acting on the mass m and fully describes the rotation by means of the second Newton's law. As told, A presents inertial observer.

Observer B, considered by Huygens, does not observes mass m at rotation but states equilibrium of the ball at rest. Tension T and the gravitational force mg are not sufficient for this state. He needs additional force –ma to reach an equilibrium. It is that additional force (-ma), not existing for Newton, but required by Huygens, that is termed today *inertial force*. Observer B, is defined as a *non-inertial observer* (the one employing inertial forces), and the rotating wheel presents a *non-inertial frame of reference.*

[28] *Ibid.* p. 1
[29] *Ibid.*

Huygens: The inertial force is similar to the force of gravity

Lacking Newton's theory of gravitation and Newton's laws of motion restricted very much Huygens in his treatment of rotation. Nevertheless, he made an ingenious guess trying to show that the gravity force is similar to centrifugal force. Were he know the formulas we know for both:

$$F = \frac{mv^2}{r} \qquad \text{and} \qquad F_g = mg$$

he would immediately see that both forces are proportional to the mass of the object and in this sense are essentially similar (they coincide in notations $a_R = v^2/R$ and $a_g = g$).

This implies that the observer B might be misled regarding the gravitational force, given that he equates the gravitational force with weight – the heaviness of the suspended body at rest (mg*). Observer A may clearly distinguish the gravitational weight (mg), but he must depart from the identity between weight (the heaviness of the body) and gravitation: in his case the centrifugal force increased the body weight. This indeed could be the point to split between gravitation and weight, but Huygens, as we already mentioned, was not there.

Totally lacking this knowledge, but being familiar with the works of Galileo and Descartes Huygens utilized their conceptions. Unlike Newton, he defined gravity using Descartes' notion of *conatus* (Fig. 2):[30]

Heaviness is a tendency to fall [Gravitas est conatus descendendi].

And then, he considered the body rotating being fastened by a rope. Here, as was mentioned, Descartes established the radial tension in the rope – conatus – rotating the stone. The intention of Huygens was to show that there is no difference between this conatus and that due to the gravity. How to do it? Huygens showed that if one frees the stone from the rope, and it recedes along the strait line (in according with Descartes' second law of motion), then, the distances from the center of the wheel increase in a sequence that was exactly established by Galileo for the free fall of an object. This testifies, Huygens thought, for the identity of the nature of rotational conatus (centrifugal force) and gravity. Here is the quote from Huygens' *De vi Centrifuga*:[31]

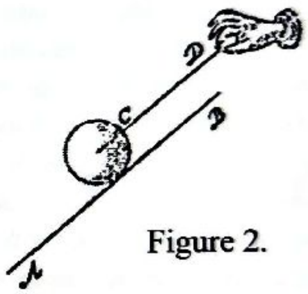


Figure 2.

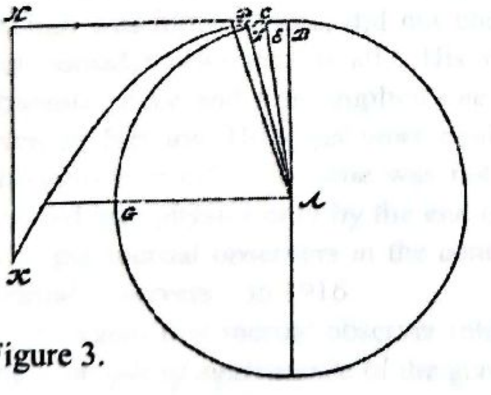


Figure 3.

Let BG [Fig. 3] be a wheel that rotates parallel to the horizon about center A. A small ball attached to the circumference, when it arrives at point B, has a tendency to proceed along the straight line BH, which is tangent to the wheel at B. Now, if it were here separated from the wheel and flew off, it would stay on the straight path BH and would not leave unless it were pulled downward by the force of gravity or its course were impeded by collision with another body. At first glance it indeed seems difficult to

[30] *Ibid.*
[31] *Ibid.*

grasp why the string AB is stretched so much when the ball tries to move along the straight line BH, which is perpendicular to AB. But everything will be made clear in the following way. Let us imagine some very large wheel, such that it easily carries along with it a man standing on it near the circumference but so attached that he cannot be thrown off; let him hold in his hand a string with a lead shot attached to the other end of the string. The string will therefore be stretched by the force of revolution in the same way and with the same strength, whether it is so held or the same string is extended to the center at A and attached there. But the reason why it is stretched may now be more clearly perceived.

Take equal arcs BE, EF very small in comparison to the whole circumference, say hundredth parts or even smaller. Therefore, the man I spoke of [as] attached to the wheel traverses these arcs in equal times, but the lead would traverse, if it were set free, straight lines BC, CD equal to the said arcs, the endpoints of which [lines] would not, however, exactly fall on the straight lines drawn from center A through points E, F, but would lie off these lines a slight bit toward B. Now it is clear that, when the man arrives at E, the lead will be at C if it was set free at point B, and when he arrives at F it will be at D. Whence we say correctly that this tendency is in the lead. But now if points C, D were on the straight lines AE, AF extended, it would be certain that the lead tended to recede from the man along the line drawn from the center through his position; and indeed such that in the first part of the time it would move away from him by the distance EC, and in the second part of the time it would be distant by the space FD. But these distances EC, FD, etc. increase as the series of the squares from unity, 1, 4, 9, 16, etc. Now they agree with this series ever more exactly as the particles BE, EF are taken to be smaller, and hence at the very outset they may be considered as if they differed nothing.

Thus this tendency will clearly be similar to that which is felt when the ball is held suspended on a string, since then too it tends to recede along the line of the string with a similarly accelerated motion, i.e. such that in a first certain period of time it will traverse 1 interval, in two parts of time 4 intervals, in three 9, etc.

Huygens' work on the centrifugal force is one of the most remarkable in mechanics of the 17th century. He preceded Newton's treatment of mechanics which though was more mature, did not considered non-inertial observers. In fact, he did not consider observers at all. His view of one static universe to be described in absolute space and time implied one true observer – God. Therefore, also praised, even by Newton, Huygens' work could not be truly evaluated by anybody, including Huygens himself. The time was not ripe for that. The idea of different observers entered into physics only by the end of the 19th century. Albert Einstein was the first who put inertial observers in the center of physics theory only in 1905 and the non-inertial observers – in 1916.

Huygens non-inertial observer rotating with the wheel was actually able to declare the *principle of equivalence* of the general theory of relativity: [32]

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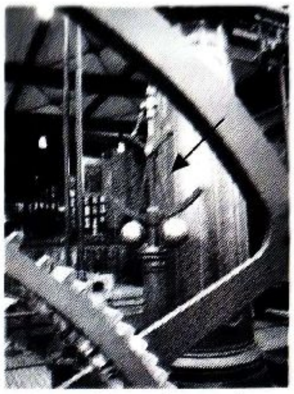
[32] Giancoli, D. C. (1988). *Physics for Scientists and Engineers*. Englewood Cliffs, NJ: Prentice Hall, p. 155.

There is no experiment observers can perform to distinguish whether an acceleration arises because of a gravitational force or because their reference frame is accelerating

One may alternatively claim that this indistinguishability manifests the universal for all matter equality (or proportionality) between inertial and gravitational masses.

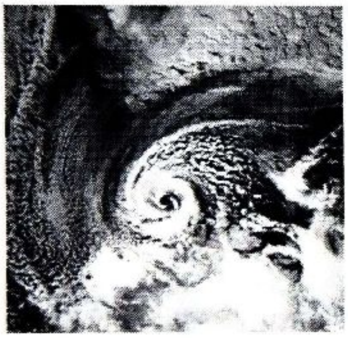
Reality of inertial forces

Huygens opened the door to the new physics, but nobody entered that door at his time, neither Newton, nor anybody else, including himself. Seemingly, there was a need first to go and construct physics without observer (or, more exactly of the unique observer) as Newton did. Even later in the 18th century, as shown above, when d'Alembert in 1758 manipulating with Newtonian equation obtained the term of inertial force F_I=-ma, even then and thereafter to the 20th century, this force was considered only as a casus: *fictitious* forces, those one can manage without.



Centrifugal regulator on steam mashine

In the 19th century, physicists explained several phenomena depended on the rotation of the Earth, as observed by us, on the surface of the rotating globe – non-inertial reference system. Such descriptions essentially involved inertial forces: centrifugal and Coriolis force (the latter presents inertial force on the object moving in the non-inertial frame of reference, such as rotating Earth).



Cyclonic motion of the atmosphere

The situation flipped, now such phenomena as Foucault pendulum, cyclonic motion of the air in atmosphere and the difference in heights of banks of the rivers running along the longitude line are inconceivable without inertial forces. Now physicists ridiculed those who continue ignoring inertial forces. This is how Sommerfeld wrote in his notorious course in the 20th century:[33]

Incidentally, the operation of railroads furnishes a very vivid example of the fact that the "fictitious" centrifugal force has a very real existence. On the curve, the rail bed is banked in such a way that the outer rail is higher than the inner. The difference in height is always such that for some mean velocity of the train the resultant of gravity and centrifugal force is perpendicular to the rail bed. This procedure eliminates not only the danger of overturning about the outer rail, but also a harmful unequal loading of the rails.

Summary

Our excursus to the history of "the force of inertia" revealed a complex and impressive conceptual story. In the ideas on inertia of Kepler, Newton, Euler, d'Alembert, Newton and Huygens we observed different conceptions. Summarizing

[33] Sommerfeld, A. (1952). *Mechanics. Lectures on theoretical physics*. Vol. 1. New York: Academic Press.

these ideas could be helpful to understand the idea of inertia – one of the central in physics:

Kepler introduced inertia and considered it as a passive resistance to motion (sluggishness).

Newton conceived inertia as an active internal force with which the body preserves its state of motion or rest and resists to its change. He also distinguished between weight (gravitational force) and inertial mass.

Euler refused to consider inertia as a force and claimed it to be a faculty of the body by which it maintains its state of motion or rest. He also introduced the new form of the second Newton's law (used in introductory courses) F=ma, in which inertia entered as inertial mass m.

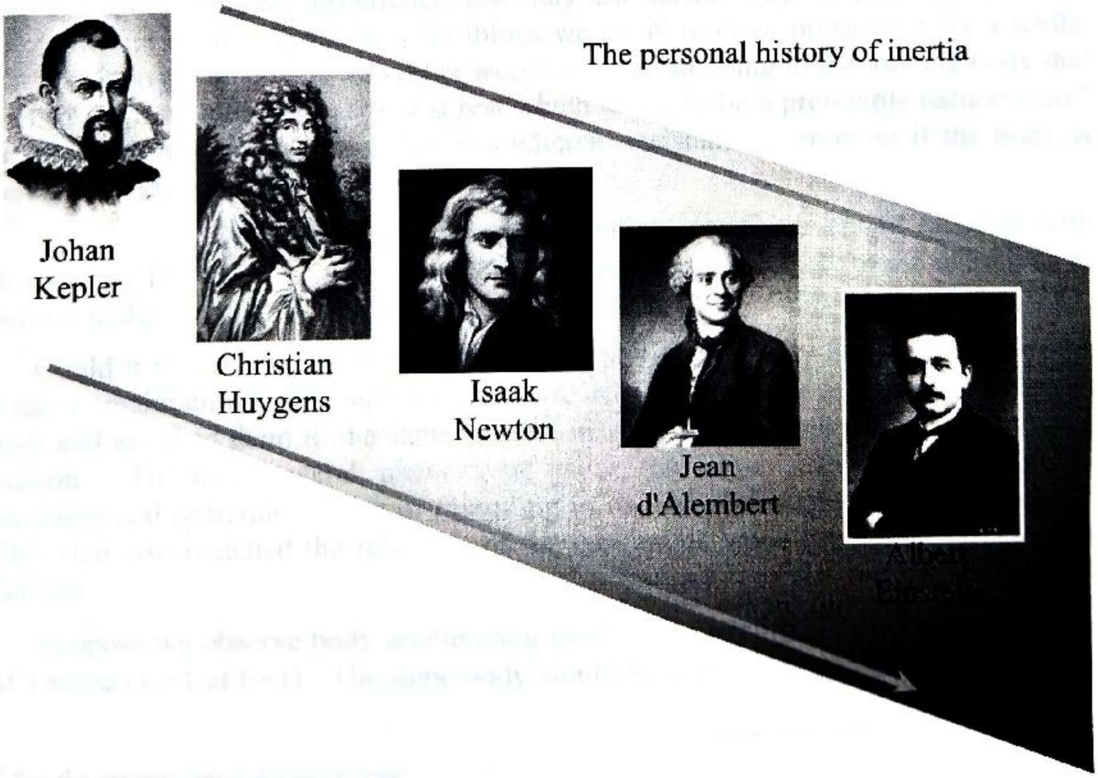
Huygens introduced the inertial (centrifugal) force and revealed its reality while considering rotating (non-inertial) observer. He obtained its functional dependence of the centrifugal force

$$F_{c.f.} = \frac{mv^2}{r}$$

and demonstrated the essential similarity in nature between centrifugal force and gravity.

D'Alembert obtained the general form of inertial force, $F_I = -ma$, but this step was considered as a trick by which any dynamics problem is reduced to statics, instead of considering it as a legitimate description by the non-inertial observer moving with the frame of reference accelerating at the rate a.

Einstein, by framing the modern physics by multiple observers, legitimized the inertial forces (in the form of d'Alembert) as real forces (as was the centrifugal force for Huygens), that is, defined operationally by the non- inertial observer.



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Historical and philosophical background including nature of science

The nature of science is such that it develops gradually and as a whole presents a cumulative product of human making sense of reality and revealing the principles of its organization. In the course of history scientific revolutions occur when the accumulated tension between the unexplained by the old theory and the new ideas causes a breakthrough, introducing a new picture of the world and replacing the old one. However, a close look may discover that the new theory bears significant traces of the previous ones. This thought once surprised Heisenberg, when he heard it from Einstein. The idea that the new theory is inevitably linked to the old one could be illustrated by the story of inertia. In any case, the knowledge of conceptual roots in science occupies the history and philosophy of science and promotes a genuine understanding of science on behalf of students and teachers.

Impetus – a conceptual predecessor of inertia

A close look may reveal that inertia concept bears the imprint of the central concept of earlier concept of impetus. The latter was central in the medieval physics of motion[34]. In the course of the scientific revolution of the 17th century, impetus was criticized and refuted due to the efforts of Galileo and Descartes. However, the idea of a sort of "charge" of motion inside the body that puts it in motion is very close to the common sense of a layman who expects that motion is supported by certain faculty ("fuel"). As such it often presents the initial knowledge with which individual begins his/her learning about physics of motion. Inertia of the body – is a philosophical idea by which we explain the fact that the body preserves its state of motion or rest. This idea presents a more mature concept that replaced impetus in the modern physics after 17th century.

Two faces of inertia

Basing on the everyday experience, one may ask herself why it is difficult to move things? And another question, why things we set in motion, preserve it for a while, after we stop pushing them? In other words, is there anything in the moving body that prevents it from stopping, getting at rest which seems to be a preferable natural state? Aristotle did not ascribe to bodies any inherent resistance to motion: if the body is pushed, it moves in proportion to the push.

Kepler, who held Aristotelian views on motion, was however not satisfied with this vision. He was the one who ascribed to body inertia, but only to the resistance to motion under the push or pull from the external mover.

Could it be that that something that resists putting the body in motion is also the cause to maintain it? Although it might look as different phenomena, Newton united both and ascribed them to the same cause – inertia, the force resisting any change of motion. To unite several phenomena under the same cause was one of his philosophical principles "rules of reasoning in natural philosophy" that he followed. This step also matched the relativity of the concept of velocity and the principle of Galileo.

Suppose we observe body accelerating from the rest ($v_1=0$ at $t_1=0$) to the velocity of 1m/sec ($v_2=1$ at $t_2=1$). The same body would be seen to the observer moving with

[34] See the correspondent historical case.

the velocity V=1m/sec in the same direction as decreasing its velocity from $v'_1=-1$ (at $t_1=0$) to $v'_2=0$ (at $t_2=1$). In accord to the principle of Galileo there cannot be any conceptual difference between the two pictures in which for the first observer the body accelerates from the rest to motion and in the other – decelerates from the motion to rest. So, as long as we go with the physics of Galileo's relativity there must be unique inertia.

The progress made by Newton regarding inertia might remind recognizing by people of the ancient world that "evening star" or "morning star" were actually the same object, the planet of Venus, that being closer to the sun may appear to us only in its vicinity at sunset or sunrise.

Evolution of the role of observer

In introductory physics course students learn that the framework of Newton's laws of motion is valid for describing reality *only by inertial observers*. After clarifying the status of inertial forces it became clear that introducing inertial forces allows the non-inertial observers to apply the same framework. This expanding of the area of validity, the ability to serve any observer, represents the nature of science which seeks the most inclusive framework of universal and objective account for natural phenomena.

In general, one may identify three important periods in the perception of the role of observer:

I. The physics of Aristotle. By considering the cosmos to be spherical, finite, static and geocentric the single immobile observer describing the whole cosmos was the only one relevant for physics.

II. The classical physics of 17-19 centuries. Although Newton stated the space to be absolute immobile reservoir of the world (in a sense preserving Aristotelian conception), the very fact that the universe was considered to be *infinite* and governed by the principle of Galileo equating all possible inertial observers made such observers the only legitimate for physics laws to be applied.

III. The modern physics (from the 20th century). Following the vision of Einstein non-inertial observers were made also legitimate for physical description, given the existence of inertial forces. Thus any observer is legitimate for the description of the world by means of objective laws of physics.

Furthermore, the discovery of the principle of equivalence, which states identity of the force of inertia to the gravitational force for the non-inertial observer, opened the modern understanding of weight of the body and was directly linked to the forces of inertia. Non-inertial observer is currently legitimate in physics exactly like inertial observer.

Expanding the concept of force

The history of inertial force touches on the meaning of the concept of force in general. It was Newton who placed the concept of force in the center of the new paradigm of classical mechanics: the universe was considered empty space with point masses interacting in pairs by means of central forces of gravitation. In this paradigm, force is exerted by one particle on the other on the line connecting between them.

However, his definition of external (impressed) force was more inclusive:[35]

An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line

This definition does not restrict force to be central or interactive (as gravitational force) but any factor causing the change of state of rest or motion. Perhaps this is not fair to expect from Newton this subtlety at the time when all known interactions (gravitational, static electricity and magnets) worked at straight line between the agent and subject. However, although the discovered by Oersted effect of an electric current on a magnetic needle in 1820, and the magnetic force on a charge introduced by Maxwell in 1861 were not central, they still matched the Newtonian requirement of force: causing the change of state.

The requirement of operational definition for physical quantities was recognized within the positivistic philosophy of science starting from Mach[36], Kirchhoff and Hertz at the end of the 19th century. The apex of success, however, this approach obtained in the theory of special relativity when applied by Einstein to the definition of simultaneity. The requirement for operational definition for any physical quantity was put to the basis of the special philosophical trend – operationalism.[37] Later this requirement was moderated by Margenau,[38] by demonstrating the necessity of both nominal (theoretical) and operational definitions for each concept in physics. Ever since, the importance of operational definitions was widely recognized in physics.

After this introduction, the problematic of inertial force becomes clear. On one hand, the inertial force is not interactive (one cannot point to the pair: action-reaction forces and two objects interacting by means of this force). On the other hand, inertial forces are directly measured (cause the change of state of motion) in the perception of a non-inertial observer. Seemingly, if one recognizes the legitimacy of non-inertial observers (as is common in modern science), this implies the recognition of the inertial forces as real forces.

On the nature of the scientific knowledge

The history of inertia illustrates the dialectical nature of the scientific knowledge. Following this history, one observes that evolution of understanding of inertia which started from qualitative descriptions and eventually arrived at formal mathematical accounts. Since then both qualitative and mathematical (quantitative) accounts for the reality are inherently interwoven and develop synergistically.

Indeed, as long as the first ideas regarding inertia by Galileo, Kepler, Descartes were solely qualitative interpretations the concept of inertia could not have impact on practical applications. Newton started with the same in his first law, but then he introduced a quantifiable concept of *inertial mass*, m and the second law, which resulted in flourishing various applications of mathematical account for physical systems.

On the other hand, the innovative application by Huygens of dynamics for the observer on a rotating wheel and obtaining the mathematical form for the centrifugal

[35] Newton, I. (1687/1999). Op.cit., Definitions, Definition 5, p. 405

[36] e.g. operational definition of mass in Mach (1893/1970). *Op. cit.*

[37] Bridgman, P. (1927/1952) *The Nature of Some of Physical Concepts*. New York: Philosophical Library.

[38] Margenau, H. (1950) *The Nature of Physical Reality*. New York: McGraw-Hill, pp. 220-244.

force, as well as the discovery by d'Alembert of the general mathematical form for the inertial force did not cause any further progress in understanding of the nature. The fact of equality of inertial to the gravitational mass was known already to Newton and was mentioned explicitly in *Principia*. However, he failed to explain it and could only express his surprise. Huygens could not use his invention beyond a straightforward application trying to explain the gravitational attraction without action at a distance, also resulted in failure. D'Alembert's finding was considered only as a trick for problem solving, and the inertial forces were labeled "fictitious" (causing a problem for curriculum designers ever since).

It all changed immediately in 1916 by the ingenious *interpretation* to the same known fact, which was provided to the very same fact by Albert Einstein in his theory of general relativity. Non-inertial reference frames became legitimate in physics.

We may imply that neither mathematical formalism nor its interpretation exclude or surpass each other in importance but their *dialectical complementarity* is required for science. The later presents the major feature of the scientific method. Paraphrasing Einstein, one may say that conceptual understanding without mathematics is empty but mathematical formalism without conceptual interpretation is blind.

On the nature of scientific concepts

The concept of inertial force may shed light on the nature of scientific concepts. The fact that inertial force "exists" only for non-inertial observers suggests a question of existence of other interactive forces. In what sense do they exist?

A discussion on this topic may lead to recognition of force (a physics concept) to be an artifact, a mental tool to describe reality. If it appears that people can imagine alternative description, forces may disappear (there is no forces in quantum theory!), similarly to the disappearance of inertial forces for inertial observer. Thus in general theory of relativity there is no gravitational force, but a curved space-time which causes the effect that we describe in classical mechanics by means of the gravitational force.

Reference to these examples in a discussion may lead to the remedy of students' naïve materialization ("reification") of abstract physical quantities such as heat, force, light, electric current, gravity, potential, etc.[39]

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Target group, curricular relevance and didactical benefit

The topic of inertia and the forces of inertia are not new in physics. Making legitimate non-inertial observers was the progress made by the modern physics already at the beginning of the 20th century. Therefore, considering inertial forces in physics class could introduce students into the ontology and epistemology of the modern physics. However, the system of education did not follow this development. The gap between science and curriculum with regard to inertial forces is already about 100 years despite of no formal complexity related to this topic. Inertia and inertial

[39] Reiner, M., Slotta, J.D., Chi, M.T.H., & Resnick, L.B. (2000). Naive Physics Reasoning: A Commitment to Substance-Based Conceptions. *Cognition and Instruction*, 18(1), 1–34.

forces are usually not included in regular and even the advanced placement school curriculum.[40]

The suggested historical excurse may convince teachers, curriculum designers, science museums, and policy makers in education to change this policy and include inertial forces into science curriculum.

Another goal for this project is to convince teachers to use history of science. We observed that teachers refrained from using historical materials:[41]

> ... *even materials produced for teachers, for example, those produced in the UK ... are not used. Attempts to produce restructured courses that put history at the center of the enterprise ... have enjoyed only marginal success, as have those that have sought to introduce a more rigorous and current view of the philosophy of science ...*

Science educators and scientists envolved in teaching also explain their negative attitude to using HPS-based materials.[42] They usually reason by two factors:

(a) they believe that historical materials, by presenting old theories, conceptions and methods which were surpassed in the course of physics development, put astray the learner and are not relevant for studying physics today;

(b) they often state total lacking of time for including additional materials into the instruction already overloaded by the abundance of materials required by the curriculum.

Our examination, however, suggests that there could be another reason behind these two: the programs for training perspective teachers (as well as the training programs of regular practitioners in science) normally do not include any exposure neither to the history no to the philosophy of science.[43] Clearly, lacking the appropriate background, people cannot evaluate such type of materials, even if they mention different reasons for their neglecting of the history and philosophy of science.

For this very reason, our historical excurse was developed, first of all, for the teachers of physics – prospective and already practicing – showing that historical materials are worth, relevant and beneficial. We invested a special effort to expose the gradual consolidation of the present understanding through several steps made by brilliant minds who developed the concept of inertial force in successive cumulative effort. This material as a teaching unit is original and not available in other resources.

What makes using historical materials essential? Among other reasons,[44] we may mention here the specific rationale related to the inertial forces and non-inertial observers. It is a well-established understanding today that teaching science to be

[40] The situation in Israel, UK, the USA.

[41] Monk, M., & Osborne, J. (1997). Placing the History and Philosophy of Science on theCurriculum: a model for the development of pedagogy. *Science Education*, 81(4),405-424.

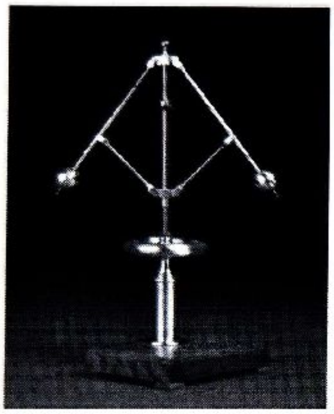
[42] Galili, I. & Hazan, A. (2001a). Experts' views on using history and philosophy of science in the practice of physics instruction. *Science & Education*, 10 (4), pp. 345-367.

[43] The situation in Israel.

[44] This issue in general terms was elaborated in the paper Galili, I. (2009). 'Discipline-culture framework of implementing the history and philosophy of science into science teaching', *Proceedings of the ESERA 7th Conference*, Istanbul, Turkey.

effective should address the existing naïve knowledge of the learners in the considered topic. This presents a central tenet of educational constructivism.[45]

The topic of inertial forces and non-inertial observers obtains its special importance because in the regular life environment, and especially in modern society, a person frequently finds oneself being in an accelerating system, getting experience of a non-inertial observer and observing such environment in media (films, news, documentary). In many real-life situations, it is simply impossible to ignore such sense experience (sense-data): in accelerated vehicles, while jumping and falling, observing astronauts in space station and many other situations. We all use not to appreciate the experience as inertial observers for its routine nature. Personal sense experience of students as non-inertial observers cast their naive knowledge organized in explanatory patterns (schemes of knowledge[46]). Those are deeply entrenched in what is called *common sense*. In class teaching, these naïve and often wrong schemes (known under the name alternative or mis-conceptions) collide with the "teacher's truth" presented for learning. This collision significantly impedes the result of learning. This is especially true regarding inertial forces, due to the rich experience in non-inertial frames of reference. Even being not instructed and even good students widely use in practice inertial forces despite of the apparent contradiction with the curriculum and the instruction they received.[47]



Technology provides another strong motivation to deal with inertial forces. Modern society is saturated with different manifestation of inertial forces: all kind of centrifuges for separation of different materials in their densities, regulators of rotation speeds (Watts' centrifugal regulator), rotational satellite (meanwhile available only in science fiction movies like "2001: A Space Odyssey" by Kubrik), etc. Each teacher for car driving uses the concept of centrifugal force to explain why one should not drive too fast on a curved road. Seemingly, inertial forces are used everywhere but in the physics class. This ignoring reality is not a recommended curricular policy in all educational approaches.

Linguistic confusion

The force of inertia is the notion which can cause confusion for its multiple meanings used by different authors. Perhaps the most striking problem is related to the impression causing on students and teachers by the titles "fictitious" and "pseudo" attached to the term force. One should remember that these terms were introduced into physics when the results of d'Alembert were obtained, but not understood for a

[45] Duit, R. & Treagust, D. F. (1998). Learning in science - from behaviorism towards social constructivism and beyond. In B. Fraser & K.G. Tobin (Eds.), International handbook of science education (pp. 3–25). Dordrecht, The Netherlands: Kluwer.

[46] e.g. Galili, I. & Hazan, A. (2000). Learners' knowledge in optics: interpretation, structure, and analysis. *International Journal in Science Education*, 22(1), 57-88.

[47] Galili, I. & Kaplan, D. (2002). 'Students' interpretation of water surface orientation and inertial forces in physics curriculum'. *Praxis der Naturwissenschaften Physik in der Schule*, 51(7), pp. 2-11

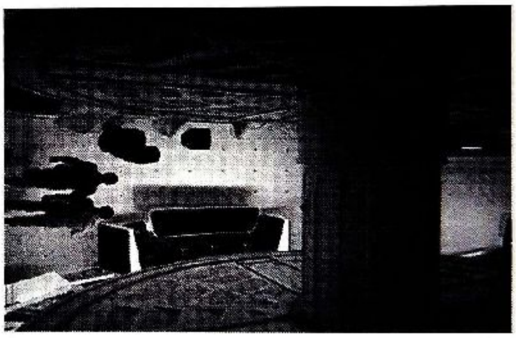
very long time, remaining merely a trick of theoretical physics.[48] It is recommended, therefore, always to check the time when this or other quote of some scientists was made. The clarification of the status of the non-inertial observers was performed only by Einstein and never before. In the modern physics, forces are defined by operational definition and therefore inertial forces are legitimate dynamic characteristics of physical situation, keeping in mind that they are real for non-inertial observers.

Another kind of confusion is possible with regard the meaning of centrifugal force. Some authors still use it in the meaning of Newton, that is to say, as a interactive partner of the centrifugal force. In this definition the centrifugal force is the force acting on the constraint (the rope, sling, that causes the object revolving around the center of motion). This use has nothing to do with non-inertial observer.

As the way to avoid confusion, it is recommended to carefully clarify the meaning of inertial force used by the particular author or lecturer. The importance of concept definitions in physics teaching was demonstrated not once,[49] illustrating physics as an exact science. The need for rigor definition became a characteristic feature of the scientific knowledge in contrast to humanities.

The relationship of inertial forces with weight

Since the gravitational force is identical to inertial force as is stated by the principle of equivalence, the operational definition of weight ("weight is the result of weighing") allows both gravitational and inertial forces equally contribute to the weight of the object. Therefore, teaching inertial forces could be related to teaching weight (including weight due to centrifugal force) and thus making learning physics more conceptually interrelated and therefore more meaningful.



Teaching about inertial force is the context to return to the operational definition of weight and facilitate it with the nominal (theoretical) definition: "Weight is the force exerted by the object on its support. It could be caused by the gravitational or inertial forces". This relationship between the two forces was foreseen by Huygens in 1659.

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Activities, methods and media for learning

Questionnaires, inquiry suggestions, role-play activities

It is very important to ask students straightforward conceptual questions to provide them with an opportunity to articulate their conceptual understanding. Among such questions regarding the subject of inertia and inertial forces, there could be the following:

- Is inertia a force? Illustrate your answer.

[48] In fact, the formal designation of F=-ma as a force of inertia was made more than one hundred years after d'Alembert by French mathematician and astronomer Charles Delaunay who presented the results of d'Alembert. We, however, may not change the tradition of ascribing this step to d'Alembert.

[49] Galili, I. & Lehavi, Y. (2006). Definitions of physical concepts: a study of physics teachers' knowledge and views. *International Journal of Science Education*, 28 (5), 521-541.

- When we set in motion the heavy revolving door of a hotel, what do we struggle with: (a) the force of gravity, (b) friction, (c) inertia? – Explain your choice and the refutation of other suggested options.
- What is the force of inertia?
- Illustrate inertial forces by concrete examples.
- Illustrate the situation you want to describe by means of inertial force.
- Why people use to name inertial force *fictitious*, or not real?
- Descartes and Huygens tried to explain gravitation by means of centrifugal force. How could it be possible if the centrifugal force by its definition directs outwards, from the center of rotation whereas the gravitational force attracts bodies inwards, to the center of attraction?
- Explain the movement of the body of a driver in a stopping (or accelerating) car.
- Small helium balloon is floating inside a stopping car. Show the inertial force exerted on it and explain the movement of the balloon in a stopping (or accelerating) car.
- It is possible to imagine a debate between Newton and Einstein regarding inertial forces. Newton would reject them and Einstein – support. Who was right in this debate? Explain.
- Huygens did not consider his centrifugal force *fictitious* but real. Feynman in his course called inertial forces "effective" (acting as a force). Hertz thought that all forces are fictions. Who was right?
- In Newton's famous experiment with a rotating bucket, water face became curved in a parabolic shape[50]. Try to explain this phenomenon with and without using centrifugal force. Which description is easier for you? Which one is correct?

Furthermore, some students may be interested in debates, defending description either with or without inertial forces. Students may prepare themselves for such a debate by reading the related materials, including the excurse above and the mentioned here questions, as well as other resources. Students may imagine their heroes as cultural and historical figures living in certain social historical environment. One of the most available means for facilitating this kind of activity is the Internet. Here are to some useful links:

http://www.rarebookroom.org/ – The "Rare Book Room" site may allow the visitor to examine and read some of the great books from the history of science.

http://gallica.bnf.fr/?lang=en – "Gallica" an online library of the National Library of France makes available works of many authors from classical Greek philosophers.

http://en.wikipedia.org/wiki/Project_Gutenberg – Project Gutenberg is the Internet's oldest producer of free access to digital books.

Among other resources of the works in mechanics, we recommend:

[50] Newton, I. (1687/1999). *Op.cit.*, Definitions, Scholium, p. 407.

- the historical overview of mechanics by Lagrange, one of the major contributors to the formulation of classical mechanics in its canonical analytical form used today in theoretical physics;[51]

- since Hertz's refutation of forces in Newton's form started from his reflection on inertial forces, getting familiar with his thoughts on mechanics could be elucidating.[52]

Discursive activity in a class

The appeal to the past is a cultural action in which knowledge appears as human product of continuous effort and cumulative effect. Culture is currently understood as dialogical by definition, and as such, it includes various ideas regardless the time the certain idea was suggested for the first time. In culture, every idea contributes. This perception presents an achievement of the philosophical thought of the 20-th century and is related to Bakhtin's dialogical approach[53] to understanding and teaching. Within it, different viewpoints on the forces of inertia establish a discourse, or dispute, which investigates each point of view.

To facilitate the class practice of this approach we isolate here the historically separated meanings of inertia and inertial forces in order to look at them in a dispute positions in a cultural dialogue to be arranged in a class. These stances are usually not recognized or distinguished in school textbooks. Here they are:

The natural philosophers of the 17-th century

> *Kepler*: Inertia is laziness (sluggishness) of all bodies by which they impede any motion, striving to remain at rest.
>
> *Descartes:* There is no inertia at all. Objects obey conservation of motion and keep it naturally, no cause is required to proceed motion.
>
> *Newton*: The inertial force is the internal force by which the body struggle with any external agent trying to change its state of uniform rectilinear motion or rest.
>
> *Hugens*: The inertial (centrifugal) force is a real force acting radially outward. They are exerted on the rotating body. This force is similar to gravity.
>
> *Euler*: Inertia is a faculty embedded in all bodies causing their ability to stay at the state of rest or continuous uniform rectilinear motion. The intention of resistance to change the state of motion manifests itself in inertial mass in the second law of motion in the form: F=ma.

Physicist of the 18-19 centuries: The inertial force is the name for the term in the equation of motion F_I=-ma. It is the force because it has the dimension of the force. It is proportional to the mass, and the latter determines the inertia in Newton's laws. Inertial force balances the active force by the external agent.

Modern technician-engineer: Inertia is the force acting on an accelerating body, that is, on the accelerating constraint (the rope of the rotating stone). It is this force that breaks the rope holding the rotating body. (Isn't that a proof that this force is real?)

[51] Lagrange, J. L. (1783/1997). Analytical Mechanics. ***Boston Studies in the Philosophy of Science***, Vol. 191, New York: Springer.

[52] Hertz, H. (1894/1956). *The Principles of mechanics presented in a new form*. New York: Dover. http://www.archive.org/details/principlesmecha00hertgoog

[53] Bakhtin, Mikhail Mikhailovich (1895-1975) was a Russian philosopher and literary critic. He is especially known in our days for his introduction of dialogism, and spatio-temporal description (chronotope) of social narratives and situations in their plurality.

Modern physicist: The forces of inertia are forces arising for the need in the description of reality by non-inertial observer (the observer in an accelerating system). Inertial forces make possible to describe reality using the same laws of motion, whether the observer accelerates (non-inertial observer) or not (inertial observer).

Expert in gravitation: The inertial force is indistinguishable (locally- in a small area) from the force of gravitation and can be explained by the curvature of the space-time in the particular location.

Although the teacher may, of course, identify him/herself with any view, it is essential in this method that students meet with multiple views. This type of teaching creates an extended *space of learning* and teaches the goal concept in variation, which proved to be effective way of meaningful learning[54].

This approach also fits to the perception of modern scientific thinking as dialogical, and so of the modern knowledge. Modern knowledge implies the knowledge of alternatives.

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Obstacles to teaching and learning

The main difficulty of the considered topic is that the forces of inertia are different from interactive forces, such as, for example, the force of electric interaction, pressure, friction, magnetic, gravitational. Although each of them is special, none but inertial is labeled *fictitious*. This presents a barrier for leaning. Indeed, history reveals the controversy related to inertial forces.

We believe, however, that physics teachers may afford a wiser, more subtle standpoint. They may suggest familiarizing and analyzing the arguments of several sides "for" and "contra" inertial forces.

The historical materials of this excurse may shake the rigidity of the extremist attitudes. The knowledge of several options (the space of knowledge) usually implies certain tolerance and plurality on behalf of the person who possesses such knowledge. In our excurse, clarification of Newton's views on inertial force, the criticism of Euler, familiarizing with Newton's meaning of centrifugal force (the force on the constraint), the formal approach of d'Alembert force, introduction of operational definition of force and evolution of the role of observer in physics – may all melt the rigidity of teachers who refrained from inertial forces and provide mature knowledge.

We may mention the observed peculiarity of those who argue for are the reality of inertial forces. They often argue by the "real effect produced by inertial forces". Thus, the rotating stone may break the sling when one increases the speed of rotation, testifying – in their view – the reality of the tension caused by the centrifugal force. Could it be merely fictitious if it causes such a clear real result?

This seems to those teachers being a cut edge argument, an ultimately convincing claim, but it has, in fact, a serious logical shortcoming. This argument states that the reason for the real phenomenon is real. The latter is true in general sense, but it implies nothing regarding inertial force. Indeed, the tension is the string is real, but in the viewpoint of an inertial observer it has another cause – the inertial mass of the speedy stone, striving to proceed along the straight line, tangential to the circle. This

[54] Marton, F. & Tsui, A.B.M. (2004). Classroom discourse and the space of learning. Mahwah, NJ: Lawrence Erlbaum.

stone at sufficiently high speed causes the thread or rope to break. It is only the rotating (non-inertial) observer who does not observe any movement of the stone, who needs "the real cause" for breaking the thread, and one finds it in the centrifugal outward force.

The main "problem" (or simply – the feature) of inertial forces is that they are not interactive (one cannot point to the second body and action-reaction partner of interaction). But is it possible that we simply do not know the partner of interaction?

Einstein revealed that inertial forces could be always interpreted as a certain gravitational field. The latter is known as a regular interactive force between bodies. This idea could be related to Mach's hypothesis (principle)[55] according to which the cause for inertial forces as resulting dynamic gravitational interaction – the interaction depending on the relative acceleration between the masses – is the matter of the distant stars and galaxies. This consideration, however, presents an open problem in physics for the complexity to evaluate interaction of this type.[56] So, meanwhile, one should better not go to the extreme negation of inertial forces also in introductory education.

In higher education, another difficulty for the learner appears with regard to inertial mass, the claim that inertia increases with the speed of the body as a relativistic effect. Although this claim is correct for inertial mass ($m=E/c^2$), it is only the rest mass which belongs to the fundamental invariant constants of matter in the relativistic physics, and the rest mass does not change regardless the speed.[57]

Among other difficulties students and teachers may face in teaching or learning the topic of inertia and inertial force one may anticipate the confusion between the mass as amount of matter (as defined in some textbooks) and mass as a measure of inertia of objects. What mass is included in Newton's second law? How is mass-inertia related with the law of inertia (the First Newton's law)?

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Pedagogical skills

Teaching about inertial forces requires from a teacher a special pedagogical skill. The difficulty of the subject is not formal but seemingly is due the conceptual complexity of inertia, the concept of force, in general, and the inertial force, in particular.

Indeed, as mentioned, the concepts of inertia and the force of inertia until today possess controversy. This means that the inertia is difficult for teaching because it requires a dialogical presenting of a dialectical view.

This touches on a more general requirement of teaching physics. Physics, like any science, is always a dialogue whether we want it or not. This is, at least, the dialogue between the teacher and student (constructivism). In practice, however, physics is rarely taught as a dialogue. Too often students' views are ignored and teaching presents univocal indoctrination. The subject of inertial forces and inertia is inherently dialogic, including various supporters and opponents, and addressing different types of observers. Presentation of this issue in class as a dialogue requires particular pedagogical skill, without which the failure is very probable.

[55] This idea comes back to the philosophizing of G. Berkeley (1726). *The Principles of Human Knowledge*. §§111–117, 1710

[56] Sciama, D. W. (1971). *Modern Cosmology*. Cambridge: Cambridge University Press.

[57] Okun, L. B. (1989), The Concept of Mass, *Physics Today*, 42 (6): 31–36.

The ability to teach dialogically is not, however, an inborn skill and can be developed. The subject of inertia and inertial forces becomes naturally dialogical in presenting historical evolution of scientific views as was done in this excurse. The important feature of such pedagogy is to present views on inertia and inertial forces, even if some of them are less justified in modern view. They all remain important for the dialogue. In fact, these vies are all related and they expand the subject on other domains of physical knowledge. This might be seen as confusing, but in fact, this is the way to match the views of different cultural personalities. This is the feature required for presenting physics in modern society.

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References for further reading

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