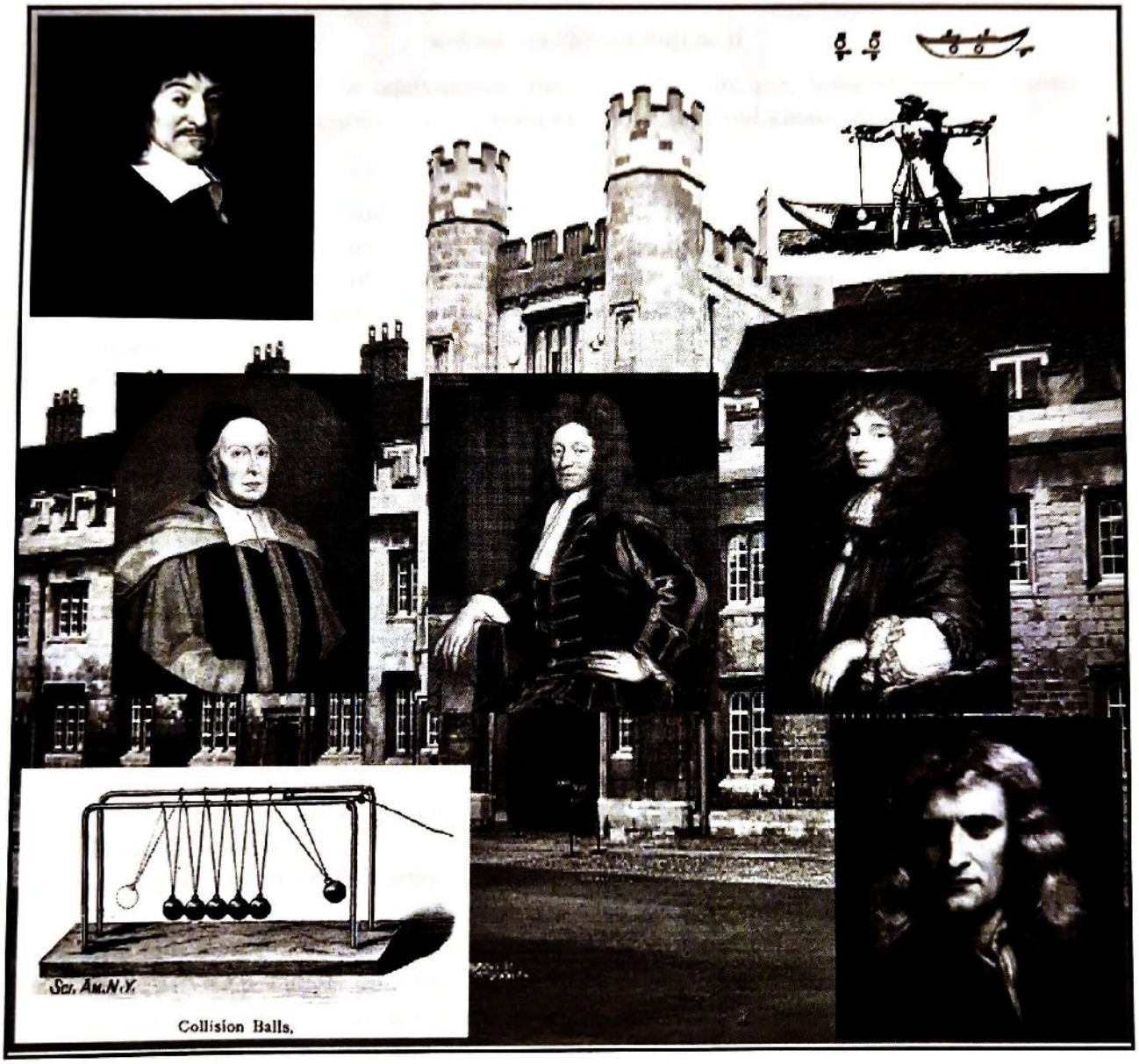
UNDERSTANDING CLASSICAL MECHANICS: A DIALOGUE WITH CARTESIAN THEORY OF MOTION



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A DIALOGUE WITH CARTESIAN THEORY OF MOTION

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Abstract

This historical case study presents the set of laws of motion established by Rene Descartes in 17th century and the inferred by him rules for collisions between material bodies. This corpus of knowledge preceded the laws of mechanics established by Newton who was inspired by Descartes' theory of mechanics. Although the laws of Descartes included new important ideas: the state of the uniform rectilinear motion as a natural state and the central role of momentum – the quantity of motion – and its conservation to describe motion of all material objects, the momentum was defined incorrectly, and the account for collisions was incorrect too. The critique of this theory and its implications to collisions was provided in the studies of Wallis, Warren, Huygens and Newton, soon after Descartes.

This historical excurse is prepared for using by teachers of physics courses in middle and high schools. Through criticizing Descartes in his erroneous understanding of such critical points as the quantity of motion as a scalar instead of vector, his ignoring the relativity principle of Galileo (the rest-motion equivalence) and neglecting empirical verification in favor of logical reasoning by certain principle this case study emphasizes by contrast these important for physics course topics. We criticized Descartes' laws of motion and considered the establishment of rules for elastic and non-elastic collisions, discovery of momentum conservation and that of kinetic energy in collisions, and finally compare the laws of motion by Descartes with those of Newton

The approach of Descartes to scientific knowledge was discussed – the rationalist philosophy of science. In contrast, it was shown the necessity of empirical knowledge although insufficient by itself too, as demonstrated by Huygens in his application of the relativity principle and discovery of the *vis viva* conservation. The promoted idea was that the nature of science presumes reciprocal support, a sort of dialectic symbiosis of rationalism and empiricism crucial to scientific progress, the need for numerical verification to validate theory and the need of theory to guide empirical research.

The case includes suggestions for beneficial activities to improve the physics curriculum by inclusion of discussion of the week and strong points of Descartes' laws of motion as a way of meaningful learning of Newton's laws of motion, discussion on Descartes' rules of collisions as a way to understand the correct account of collisions by the classical mechanics, discussion on Huygens' treatment of collisions as a way to appreciate and assimilate by students Galileo's principle of relativity – the most fundamental principle of science. These all too often escape any practical use in physics classes.

The case includes references to the research evidence of students' difficulties which could get remedy from utilizing the considered historical materials by physics teacher.

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Description of Case Study

And the demonstrations are so certain that, even if experience seemed to show us the contrary, we would nevertheless be obliged to place more faith in our reason than in our senses. Descartes

Cartesian laws of nature

Rene Descartes

It is a commonplace today to identify Newton's laws of motion with Mechanics. Indeed, these laws constitute the nucleus of the Classical mechanics. They were published in *Principia* (*The Mathematical Principles of the Natural Philosophy*) in 1687. In fact, Newton's Principia was a part of his discourse with another *Principia* (*The Principles of Philosophy*), the one by Rene Descartes published in 1644. A better understanding of the meaning of the *Principia* by Newton, one may get from the acquaintance with the *Principia* by Descartes, at least with regard to its central (for mechanics) part: the *laws of nature* and their implication – *the rules* that govern *collisions* of material bodies. In the following, we do that creating a dialogue with Descartes claims from the point of view of Newtonian mechanics.

37. *The first law of nature: that any object, in and of itself, always perseveres in the same state; and thus what is moved once always continues to be moved.*

Indeed, from the same immutability of God can be known certain rules or laws of nature, which are the secondary and particular causes of the diverse motions that we perceive in individual bodies. The first of these is that any object, insofar as it is simple and undivided, remains, in and of itself, always in the same state and is never changed, unless by external causes. Thus, if some part of matter is square, we may easily persuade ourselves that it will continue perpetually to be square; unless something should come from elsewhere that changes its shape. If it were at rest, we do not believe it would ever begin to be moved, unless it was impelled to do so by some cause. Nor is there any greater reason, if it were moved, why we should think that it would ever of its own accord, and impeded by nothing else, interrupt its own motion. And therefore one should conclude that that which is moved is, in and of itself, always moved. But, because we are here talking about the earth, the constitution of which is such that all motions that take place near to it are shortly halted, and often due to causes that are hidden from our senses, we have often from earliest times judged that these motions, which were so halted by causes unknown to us, cease of their own accord. And then we are inclined to posit of all what we seem to have experienced in many, namely that these [motions] by their nature cease, or tend toward rest. Actually, it is wholly in opposition to the laws of nature; for rest is contrary to motion, and nothing can be moved to its contrary, or to its own destruction, by its own nature.

Reflection. Descartes stated here, albeit for the reason we might not share (the "immutability of God"), a very important principle – the need for external influence to change the state of a body. This idea was new in the sense that Descartes addressed motion as a state of a body (not a process) and thus introduced the idea of inertial

motion: motion without mover – the principle of the new physics we call classical mechanics.

At the same time, we cannot agree with Cartesian opposing of motion to the rest. This is the old Aristotelian conception and we do not accept it any more. The rest-motion opposition matches naïve intuition (often addressed as "commonsense"). It is this idea that hinders understanding of inertia as something that requires a special agent – a mover.

Our experience testifies that any two rectilinear and uniform movements with arbitrary velocities are indistinguishable, and the state of rest could be one of them. Motion and rest coexist in one body by mere relation to different objects. This presents Galileo's principle[1]. In fact this fact implies *the law of inertia*: the state of movement that does not require any power for its preservation, whether rest or motion. As we saw, although Descartes agued for the principle of inertia (as preserving the state until intrusion of any external cause), he still kept the difference between motion and rest and considered them as opposites. In addition, the claim that "nothing can be moved to its contrary, or to its own destruction, by its own nature" is ambiguous and obsolete. As we know today, atoms transfer spontaneously from one state to another, and atomic nucleus decay spontaneously, transferring from one element to another without any external intrusion.

38. *On the motion of projectiles*

Certainly, everyday experience of things that are thrown wholly confirms our rule. For there is no other reason why thrown [bodies] should continue in motion for any time after they have been separated from the thrower than that once moved they continue to be moved, until they are slowed by contrary bodies. And it is manifest that they usually are gradually retarded by the air, or some other fluid bodies in which they are moved, and hence their motion cannot last long. For we can experience air resisting the motions of other bodies by our sense of touch if we strike it with a fan; the flight of birds also confirms the same thing. And there is no other fluid which does not, even more manifestly than air, resist the motions of projectiles.

Reflection. From his first law of nature Descartes deduces the answer to the question that was difficult for Aristotle to answer: why the stone continues to move after it leaves the hand of the person. Aristotle suggested a special mechanism of air turbulence – *antiperistasis* – which preserved the continuous pressure on the moving stone by the air turbulence. This mechanism was criticized and refuted by many scholars who argued by counterexamples, such as a rotating top or an arrow very sharp at its both ends. In contrast, Descartes' new principle of preserving the state of motion removed the need for antiperistasis. At the same time, Descartes, although worked after Galileo, did not provide a detailed account for the projectile motion as Galileo did (trajectory, velocity change, acceleration), and remained on the very general level of understanding. This approach was helpless to explain and precisely describe the motion of projectiles quantitatively, as classical mechanics requires.

39. *<u>The second law of nature</u>: that every motion of itself is rectilinear; and hence what is moved circularly tends always to recede from the center of the circle it describes.*

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[1] The state of the rest appears in *Discourses on Two New Sciences* published in 1638 by Galileo Galiley. Galileo considered rest as a state of motion with infinitely low speed. This way rest lost its special status.

The second law of nature is that any part of matter, considered apart, never tends to continue to be moved along any oblique lines, but only along straight lines, even if many are often forced to deflect due to the collision of others, and, as has been said shortly before, in any motion a circle is somehow made from all the matter moved at the same time. The cause of this rule is the same as that of the one preceding, namely the immutability and simplicity of the operation by which God conserves motion in matter. For He does not conserve it other than precisely the way it is in the moment of time in which He conserves, with no relation to what perhaps was shortly before. Although no motion occurs instantaneously, it is nevertheless manifest that everything that is moved, in the single instants that can be designated while it is moved, is determined to continue its motion toward some direction along a straight line, and never along any curved line.

For example, stone A, rotated in sling EA around circle ABF, at the instant in which it is at point A is determined to motion in some direction, namely along a straight line toward C, such that the straight line AC is tangent to the circle. But one cannot arrange that it be determined to any curved motion; for, even if it previously came from L to A along a curved line, nevertheless nothing of this curvity can be understood to remain in it when it is at point A. This is also confirmed by experience, because if it then left the sling it would not continue to be moved toward B, but toward C. From which it follows that everybody that is moved circularly, perpetually tends to recede from the center of the circle it describes. We experience this by tactile sense in a stone that we move in a circle with a sling. . .

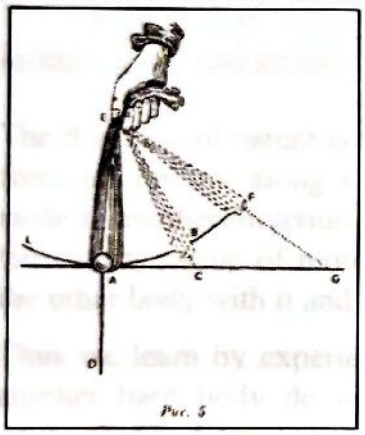


Fig. 5

Reflection. What is different in this law, compared with the first law of motion stated above? Here Descartes refined the previous claim of motion preservation: not any motion is preserved, not rotation, for example, but only rectilinear one. We still miss here "uniform" for the motion that is also required. However, let's not forget that even Galileo used to think about *rotational* inertial motion. Descartes explicitly rejected this idea; he provides the rectilinear motion with a special status: only such motion is preserved as inertial. Again he reasoned by his major rationale – God's nature. This claim, however, sounds arbitrary for us and may justify any nature of motion. In fact, it might be sufficient for physics that experience confirms the special status of the rectilinear uniform motion.

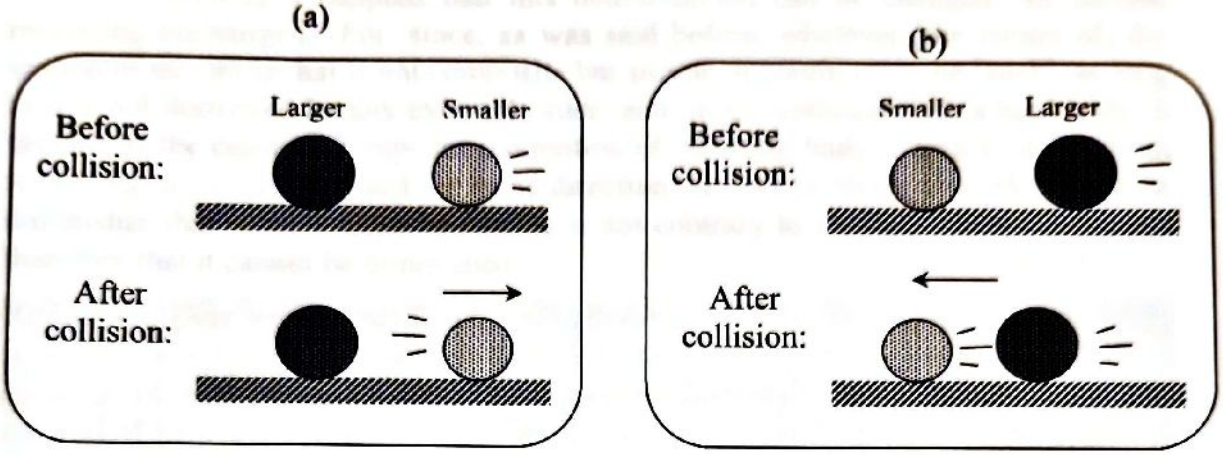
Another question: What is different in this law (the second law of Descartes), compared with the First Law of mechanics as commonly taught in physics class?

Descartes' law is different. It doesn't address the situation with no force active. Descartes addresses the tendency of the body which is compelled to move on a circular path to proceed in the tangential direction which is realized the moment that the agent imposing the circular motion (the rope) ceases to influence the stone. This Cartesian approach – considering tendency - was adopted also by Newton who in the same spirit stated in his *Principia* in 1687 (Newton 1687/1999):

Every body perseveres in its state of resting or moving uniformly straight on, except inasmuch as it is not compelled by impressed forces to change that state.[2]

As we see, Newton does not talk simply about the absence of forces but, similar to Descartes, rather about the tendency to preserve the state of resting or moving uniformly which succeeds *inasmuch as the external force diminishes.*

40. <u>Third law</u>: that a body, in colliding with another larger one [Fig. a], loses nothing of its motion; but, in colliding with a smaller one [Fig. b], loses as much as it transfers to that one.



The schematic representation of stated law of collisions. This is further refined in the following text and seven rules of collisions

The third law of nature is this: where a body that is moved meets another, if it has less force to continue along a straight line than the other has to resist it, then it is turned aside in another direction [Fig. a], retaining its quantity of motion, and changing only the determination of motion. If, however it has greater force [Fig. b], then it moves the other body with it and loses as much of its motion as it gives to that other.

Thus we learn by experience that any hard bodies that, when thrown, strike against another hard body do not therefore cease from motion, but are reflected in the opposite direction. On the contrary, however, when they meet a soft body, to which they can easily transfer all their motion, they immediately come to rest. All the particular causes of the changes which occur in [the motion of] bodies are contained in this third law, or at least those that are physical; for whether, and in what way, human or angelic minds have the force to move bodies, we do not now inquire but reserve for our treatise *On Man.*

Reflection. Descartes stated here a central law in physics related to his name ever since – the conservation of the quantity of motion, or in our terms, momentum. This law, however, is refined to two cases: a collision with smaller or bigger body, which is obscure and ultimately erroneous. We will discuss this law in the following. This law purports to account for any collision of corporeal hard bodies. Descartes correctly separates between two essentially different cases: collisions of hard and soft

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[2] Newton, I. (1687/1999). *The Principia. Mathematical Principles of Natural Philosophy*. Translated by B. Cohen & A. Whitman. University of California Press, Berkeley, CA. In the Academic translation into Russian (by Krilov) the first law appeared in a more inclusive form:

Every body *continues to preserve* its state of rest or uniform motion in right line *until it is* and *so far as* it is *not* compelled to change that state by forces impressed upon it.

bodies; each implies a different account. This dichotomy of collisions was preserved in physics. The claim regarding soft bodies is apparently wrong: inelastic collision does not imply stopping for the colliding bodies. Seemingly, Descartes kept in mind dissipation of motion in motion within the "soft" medium, such as when a hard ball rolls into sand. As appropriate for a scientific text, Descartes deliberately excluded all non-physical cases.

41. Proof of the first part of this rule.

The first part of this law is demonstrated on the basis that there is a difference between motion considered in itself and its determination in a certain direction, by which [difference] it happens that this determination can be changed, the motion remaining unchanged. For, since, as was said before, whatever [the nature of] the motion of any thing that is not composite but simple, it continues to be [such], as long as it is not destroyed by any external cause; and, in the collision with a hard body, it appears as the cause that impedes the motion of the other body, which it meets, from remaining determined toward the same direction, but not [a cause] that takes away or diminishes that motion, because motion is not contrary to motion, whence it follows therefore that it cannot be diminished.

Reflection. Here we see that Descartes in defining the quantity of motion completely separates between motion (its quality) and its determination (direction). He states that collision of *hard* bodies may influence determination of the motion, but not the amount of motion, arguing: "motion is not contrary to motion". In reality, however, one may observe inelastic collision of two equal *soft* balls meeting at equal speeds but in opposite directions. After the impact, the balls may stop completely, being stuck together. This shows that one cannot talk about motion in terms of a positive number, momentum must have both magnitude and direction. Introduction of momentum as a vector removes the ambiguity of "big" and "small" bodies, "fast" and "slow" movements. For us (apparently not for Descartes!) motion itself is only a relative quantity: its magnitude depends on the chosen frame reference implying equivalence of motion and rest as stated by Galileo.

42. *Proof of the second part.*

Furthermore, the second part is demonstrated from the immutability of the operation of God, now continually conserving the world by the same action by which He formerly created. For, since all things are filled with bodies and, nevertheless, the motion of any body tends in a straight line, it is most clear that, from the beginning, God, in creating the world, not only moved its various parts in different ways but at the same time also brought it to pass that some would impel others and transfer their motions to them; in order that now, in conserving that [world] by the same action and by the same laws by which He created, He conserves motion not always fixed in the same parts of matter but passing from some parts into others according as they collide with one another. And thus this continuous change of things created is itself to be argued of the immutability of God.

Reflection. Here Descartes explains that he insists on the conservation of motion because it is reduced from his major principle: the immutability of God. Is this necessary or convincing? This claim of Descartes is metaphysical. It characterizes Descartes' beliefs about the nature of science: his science is deeply interwoven with

the rational philosophy – removed by the scientists later as metaphysical.[3] Descartes wanted to reason the conservation of the quantity of motion (momentum). This reason was found much later, in the beginning of the 20th century by Emile Notter, who shoed that the conservation laws are results of the continuous symmetry of the system. In the case of momentum this is the symmetry of translation in space. Newton, unlike Descartes, had a hard time trying not to include metaphysical ideas into physics even when he could not provide any material mechanism for the reality he described. Thus Newton described the gravitation but eschewed explaining how it "works". This did not prevent him from the great success in establishing physical account for fundamental phenomena in the nature.

Understanding of collisions

After presenting the general laws of nature Descartes in his *Principles* proceeded to their refinement, especially of the third law of motion conservation. He made it in a set of *rules* which had to explain the collisions between the material bodies. He united them under the title:

How much the motion of any body is changed by the collision of other bodies?

46. ***First rule.***

First, if these two bodies, say B and C, were wholly equal and were moved equally fast, B from the right toward the left and C on a line with it [*illi in directum*] from the left toward the right, when they collided with one another, they would be reflected and afterward would continue to be moved, B toward the right and C toward the left, no part of their speed having been lost.

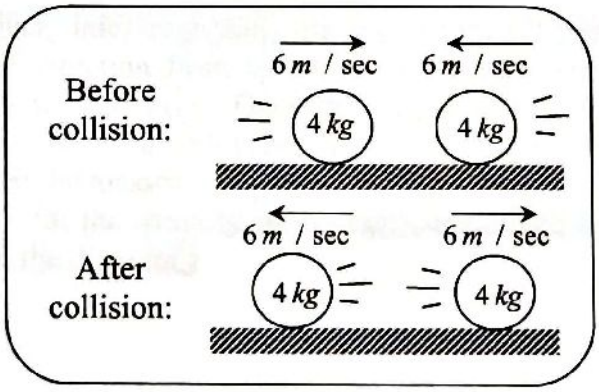


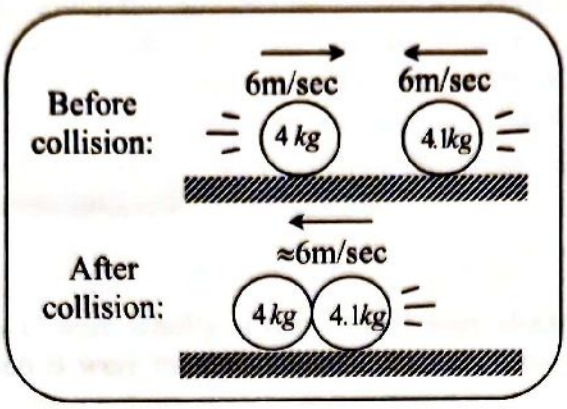
Illustration with arbitrary figures

Reflection. Think about the following questions: What could be the arguments of Descartes supporting the rule? Is this rule consistent with the previously stated laws? Does this rule fit out experience? Under what circumstances? (define "hard" and "soft" bodies)

[3] The most famous among those scientists was Ernst Mach who rewrote classical mechanics in his *The Science of Mechanics* (1889), in order to remove from it the metaphysical claims, that is to say, those which were not a subject of empirical verification. This was a new philosophical approach to science of the trend of thought termed later *logical positivism*.

47. *Second rule.*

Secondly, if B were just slightly [*tantillo*] larger than C, other things being posited as before, then only C would be reflected, and both would be moved toward the left at the same speed.



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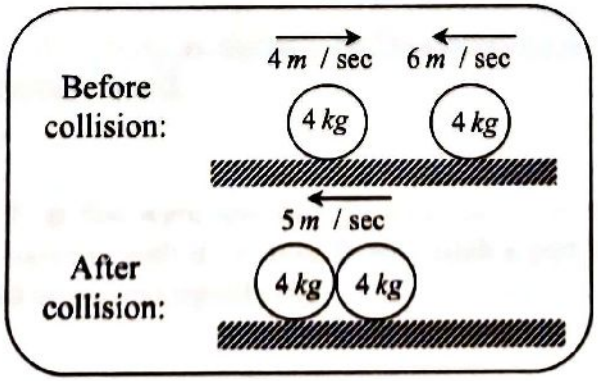
Reflection. Descartes predicts that after such a collision the two hard bodies will move to the left, that is to say, the total "victory" of B is guaranteed by a however small fraction of mass. This is not what happens in reality.

Indeed, in the case of two equal bodies (rule 1) Descartes stated that they returned in opposite directions with equal speeds. But if rule 2 were correct, we would never observe the cases of bodies receding after collision, since in practice there are no absolutely equal bodies (equal number of atoms!): one body is always "slightly" bigger than the other. Therefore, if rule 2 were correct regardless the smallness of mass difference, we would never see two bodies recede after collision. This is not what happens…

One may, however, infer regarding the framework of Descartes account. He completely separated direction from speed and thought in terms of scalar (always positive) quantity of motion (mv). Once the quantity of motion is greater in B it overcomes the motion of C and both proceed to the left. They continue almost at the same speed (the closer the masses are) preserving the quantity of motion as he defined it. (Write the balance for the quantity of motion – as defined by Descartes – for the magnitudes shown in the diagram.)

48. *Third rule.*

Thirdly, if they were equal in mass, but B were moved just slightly faster than C, not only would both continue to be moved toward the left, but also the half part of the speed by which C is exceeded by B would be transferred from B to C. That is, if before there were six degrees of speed in B and only four in C, after mutual collision each would tend toward the left with five degrees of speed.



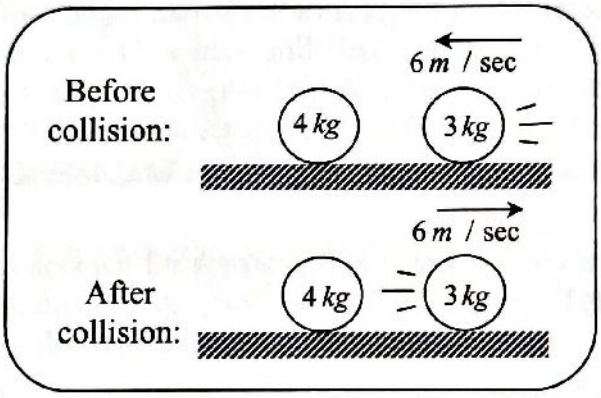
Reflection. This is again a wrong statement. This time, Descartes provides exact numbers displaying his account. One may argue by continuity: there can be no drastic change from rule 1 when B moves only slightly faster. If rule 1 holds, after the collision the bodies should recede with only "slightly" changed speeds (B a bit slower than C).

Let's speculate about the possible origin of Descartes' thought. If the quantity of motion is defined as a product of mass and speed mv (no direction), then the higher quantity of motion of body B provides the victory, meaning for Descartes that both will move to the left. Now, to preserve the quantity of motion, both bodies will move with the mid speed, which is 5m/sec (masses are equal) to preserve the quantity of motion. This implies they will move together.

49. *Fourth rule.*

Fourthly, if body C were wholly at rest … and were slightly larger than B, whatever the speed at which B were moved toward C, it would never move this C, but would repelled from it in the contrary direction; because a body at rest resists a greater speed more than a smaller one, and this in proportion to the excess of the one over the other, and, therefore, there would always be a greater force in C to resist than in B to impel.

Illustration with arbitrary figures



Reflection. This understanding of Descartes is amazingly odd to classical mechanics where if C remained at rest and B was reflected, the total momentum would not preserve. The problem is apparent: Descartes believes that motion and rest are essentially unequal – a great misconception of the old mechanics starting from Aristotle and removed by the principle of Galileo.

Within his vision, Descartes suggests a resisting force of the *greater mass* at rest to the motion of *lesser mass* wins! Notice also the violation of action-reaction symmetry (the third Newton's law) the resisting force increases in magnitude with the increase of speed of B and surpasses it in preserving the state of rest of body C after the impact. This goes with the intuition: a greater force is required to stop the faster body. The quantity of motion, as defined by Descartes, is preserved when body B returns back with the same speed.

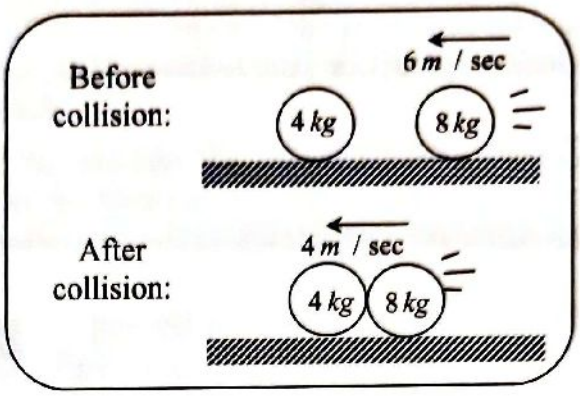
50. *Fifth rule.*

Fifthly, if the body C at rest were less than B, then, however slowly B were moved toward C, B would move C with it, by transferring such a part of its motion to C that afterward both would be moved equally fast.

That is, if B were twice as large as C, it would transfer to it the third part of its motion, because that one third part would move C as quickly as the two other remaining [parts would move] B [which is] twice as large.

And thus, after B had collided with this C, it would be moved a third part more slowly than before, i.e. it would require as much time to be moved through a distance of two feet as before to be moved through a distance of three. In the same way, if B were three times as large as C, it would transfer to it the fourth part of its motion, and so on.

Illustration with arbitrary figures



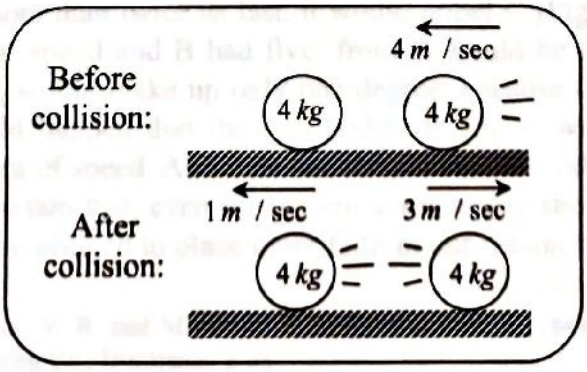
Reflection. In this case too the *greater mass* wins and imposes its state (this time, of motion) to the *smaller mass*. Descartes still cares about the conservation of his quantity of motion (mv), which in this case (one direction of motion!) coincides with regular (modern) momentum. Indeed, with the numbers of the figure, we see that before the collision $(mv)_{tot}=8\times6$, and after: $(mv)_{tot}= (4+8) \times4$, That is the quantity of motion preserves.

In classical mechanics, to account for elastic collision, one takes care of conservation of momentum as well as kinetic energy, not known to Descartes. This implies that the result of Descartes (and so this whole rule) is totally wrong.

Importantly, if one applies the relativity principle (also ignored by Descartes, who worked after Galileo's publications) this rule contradicts rule 4. Indeed, consider yourself sitting on B. You will observe B at rest while C approaches B. In accord with rule 4 body B should remain at rest and C repel (not moving together with B as stated in rule 5).

51. ***Sixth rule.***

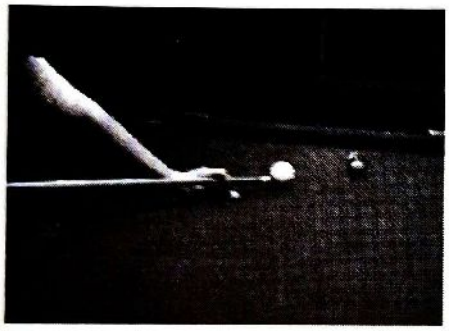
Sixthly, if body C at rest were most accurately equal to body B moved toward it, it would be partly impelled by B and would partly repel it in the contrary direction. That is, if B were to approach C with four degrees of speed, it would communicate to C one degree and with the three remaining would be reflected in the opposite direction.



Reflection. This rule is puzzling but could be interpreted[4] as the combination of two previous rules. Since C is neither larger nor smaller than B there will some tendency for C to react as if it were smaller than B, and for both to move with a speed of two (rule 5). However, there will be an equal tendency for C to behave as if it were larger and hence for it to acquire no speed whatever (rule 4). Taking the average of these two tendencies gives the result. For body B speed it is: $\frac{4+2}{2}=3$.

The mentioned by Descartes result again demonstrates (due to the figures provided) that Descartes conserves his "momentum" – the product of mass and speed – and not ours – the product of mass and velocity of motion. His conservation means: $4\times4 = 4\times3 + 4\times1$ (directions ignored).

Again Descartes violates relativity: the case of rule 6 equivalent to the case of rule 1 if one moves at the speed 2 in the direction of body B, leading to the general statement: whenever two equal masses collide elastically they exchange speeds and directions, that is, velocities.



But the most amazing fact is that just this case, considered in rule 6, is known to anybody just once playing with hard balls (billiard or similar games). The result of the impact between two equal balls, one thrown towards the second, being at rest, is that the first ball stops and the other continues with the same speed. Despite all evidence known to him, Descartes keeps with some principle which looks to him superior.

52. ***Seventh rule***

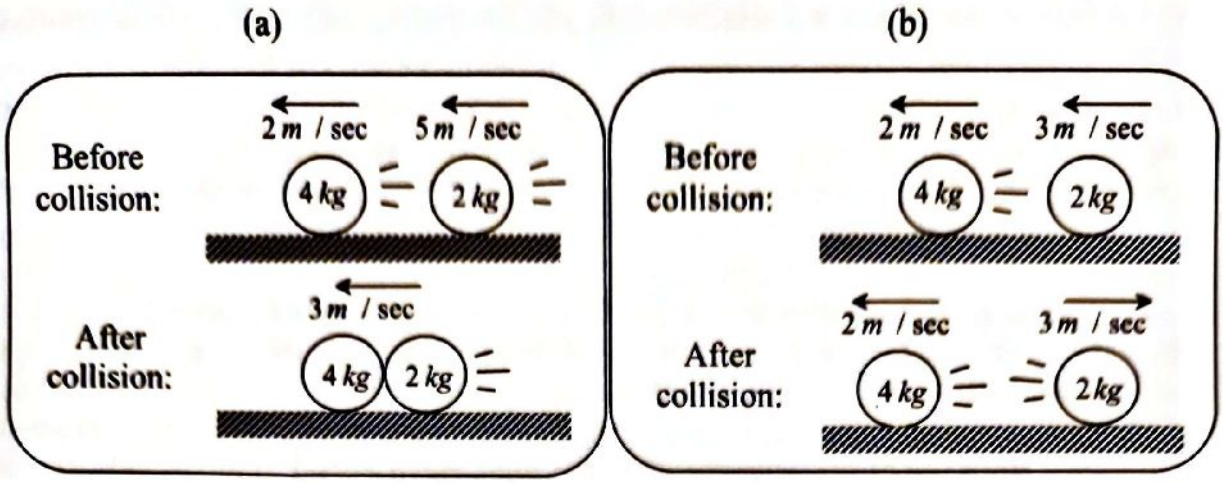
Finally, if B and C were moved in the same direction, C more slowly and B pursuing it more quickly, such that it finally reached it, and C were larger than B, but the excess of speed in B were greater than the excess of magnitude in C [Fig. a], then B would transfer so much of its motion to C that both would be moved afterward equally fast and in the same direction.

But if, on the contrary, the excess of speed in B were less than the excess of magnitude in C [Fig. b], B would be reflected in the contrary direction and would retain all of its motion.

And these excesses are thus computed: if C were twice as large as B and B were not moved twice as fast as C, B would not impel C but would be reflected in the contrary direction [Fig. b].

But if it were moved more than twice as fast, it would impel C [Fig. a]. That is, if C had only two degrees of speed and B had five, from B would be taken two degrees which, transferred to C, would make up only one degree, because C is twice as large as B. Whence it would happen that the two bodies B and C would afterward be moved with three degrees of speed. And thus one should evaluate other cases. And the demonstrations are so certain that, even if experience seemed to show us the contrary, we would nevertheless be obliged to place more faith in our reason than in our senses.

[4] The interpretation by Miller, V. R. and Miller, R. P. in Rene Descartes (1644/1983), *Principles of Philosophy*, Reidel Publishing Co., Dordrecht, p.68.



Reflection. This rule addresses a rather general case of collision between two unequal hard bodies moving in same direction with different speeds. In this case, to decide which one "wins" (that is, "stronger" or possesses "more force") Descartes compares their quantities of motion. In case (a), B won and it imposes the change the state of motion on C. Descartes preserves the total quantity of motion: $2\times5 + 4\times2 = 4\times3 + 2\times3$. In case (b), however, C is stronger and wins. This means for Descartes that C does not change its state of motion. B is compelled to change its direction, but not motion (speed) and returns back. The total quantity of motion is conserved, it looks as: $4\times2 + 2\times3 = 4\times2 + 2\times3$.

In the perspective of classical mechanics, this rule of Descartes violates both the principle of conservation of momentum as well as the principle of conservation of energy.

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The contemporaries of Descartes quickly realized that most of the rules Descartes suggested to account for collisions fail to match the reality. Descartes understood well that his rules would be criticized as not matching to the reality. This, however, did not change his mind: he trusted reasoning by chosen principle, deducing from it less general, concrete claims. If one needs to ignore empirical evidence, so it will be. In the section following the rules of impact, Descartes addressed the possible discrepancy between the observed collisions and the rules he stated:

53. *The use of these rules is difficult, for the reason that each body is touched by many at the same time.*

But, because no bodies in the world can be so separated from all the others and no bodies around us are wont to be completely hard, it is therefore the more difficult to enter into calculation to determine how much the motion of any body will be changed due to collision with others. For, one must have knowledge at the same time of all those that touch it on all sides, and these have very different effects with respect to it [*quantum ad hoc*], according as they are hard or fluid. Therefore, one must here inquire in what their diversity consists. In fact, it often happens that experience can at first seem to contradict the rules that I have just set out, but the reason for that is evident. For they presuppose that the two bodies B and C are perfectly hard, and separated from all the others in such a way that there is none around them that might aid or impede their motion; but we see no such bodies in this world.

Reflection. Here Descartes argued that the discrepancies are due to the complexity of the real environment in contrast to the ideal situations – a totally empty world – that he treated which does not correspond to the reality as it is – without empty space (no vacuum), but continuous material medium. The situation is however worse. The truth is that even in vacuum bodies would not follow the rules of Descartes but the principles of mechanics as known to us today, such as the principle of relativity, the conservation of *vectorial* momentum, and the conservation of kinetic energy in the elastic collisions. Although the factors of friction and influence of external bodies may mask the validity of these principles, this cannot save Descartes' rules of collisions. Experiments with controlled parameters demonstrate that. It is possible to monitor the influence of medium decreasing the impeding factors to any degree. The observed behavior of bodies never approached the predictions of Descartes.

The alternative theory to that of Descartes was developed very soon after his *Principles* was published. This process started from obtaining the empirical rules that govern collisions. The fundamental principles of mechanics – the law of inertia, the principle of relativity – were checked by approaching the ideal case and reasoning by extrapolation – the way physics treats the reality. The new rules led to the new theory without any reference to the metaphysical causes, so striking in the philosophy of Descartes.

* * *

Further progress and refutation of Descartes' rules of collisions

The seminal work of Descartes illustrated the interests of scientists of the 17th century to obtain a new mechanical picture of the universe. The account for collisions was in focus of this research effort since this was the most concrete model of interaction.

Unlike the natural philosophy of earlier times, and unlike Descartes, several researchers tried to get first a reliable empirical account for the quantitative rules that govern nature, whether or not they could explain them theoretically. Such was the renowned contribution of Galileo who succeeded to demonstrate in experiment that things do not just fall, but fall at constant free fall acceleration – g. To obtain a similar knowledge became the goal of several researchers who explored collisions.

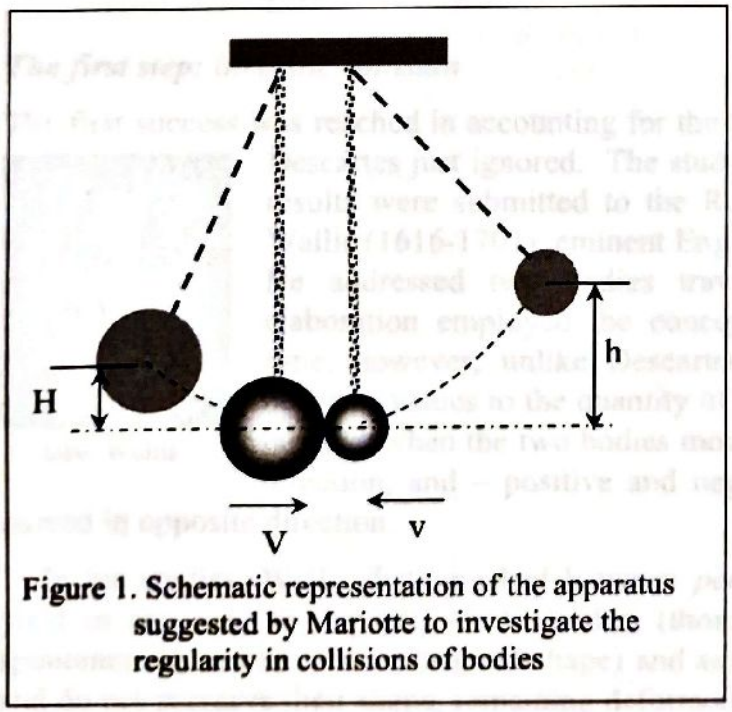

Figure 1. Schematic representation of the apparatus suggested by Mariotte to investigate the regularity in collisions of bodies

In 1668, the newly founded scientific society – the Royal Society of London – issued a call for a study which could provide a reliable account for collisions.

There was, however, a difficulty to perform such an exploration: how to measure the velocities before and after collisions? Galileo managed to infer regarding the

velocities indirectly, by measuring instead the distances of the moving bodies on the inclined plane. Collisions demanded another technique.


Edme Mariotte

We know about the elegant solution for the problem from the studies of Mariotte[5] published in 1677, in Paris. He suggested measuring the velocity of colliding bodies (m, M) by making them bobs of pendulums (Fig. 1). After Galileo it was known that the height of elevation of a thrown body is a function solely of the initial velocity and the relationship is a square proportion: $h \propto v^2$ ($v = \sqrt{2gh}$).

This relationship allowed researchers easily measure the velocity of the colliding bodies by measuring the heights they raised following the impact.

This was not enough. For the cases when one of the bodies cannot be converted into pendulum, Mariotte described an apparatus that we call today *ballistic pendulum* (Fig. 2). In this apparatus, the pendulum bob is a massive cylinder (M) which collides with a small body (a bullet - m) coming at a high speed. Again, the height that the cylinder raised (h) following the impact informs of the velocity caused by the collision is in a square proportion to the speed of the bullet: $h \propto v^2$ ($v = \sqrt{2gh}$).

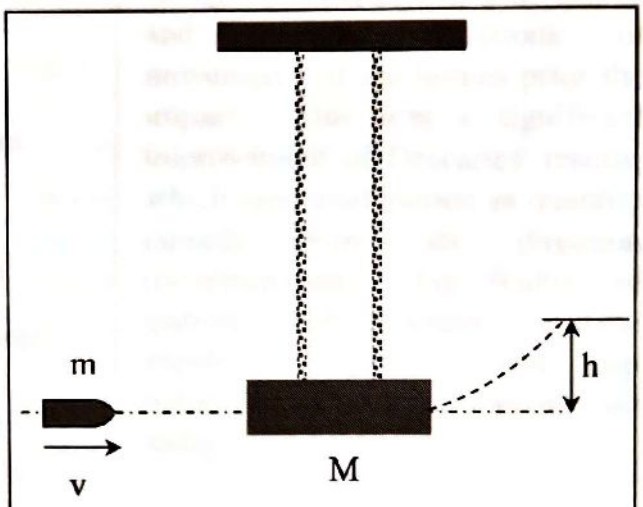

Figure 2. Schematic representation of ballistic pendulum

The first step: inelastic collision

The first success was reached in accounting for the totally inelastic collision, the type Descartes just ignored. The study was performed in 1668, and the results were submitted to the Royal Society of London by John Wallis (1616-1703), eminent English mathematician, and physicist. He addressed two bodies traveling in a straight line. His elaboration employed the concept of quantity of motion. This time, however, unlike Descartes, Wallis assigned positive and negative values to the quantity of motion mV: both velocities were positive when the two bodies moved before the impact in the same direction, and – positive and negative when the colliding bodies moved in opposite direction.


John Wallis

In his studies, Wallis distinguished between *perfectly hard* bodies (that did not yield in any way in impact), *elastic* bodies (those that yield in impact, but then spontaneously regained their original shape) and *soft* (those that deformed in impact and do not preserve their shape, remaining deformed, after the impact). The latter, in the extreme case, stick together when collide and may proceed afterwards in moving as a single body. It is just regarding the latter case that Wallis especially succeeded

[5] Edme Mariotte (1620-1684) – French priest and physicist, member of the Academie des Sciences.

and established the rule for calculating the resultant velocity (u), shared by the bodies with masses M and m that initially moved with velocities V and v[6] (Fig. 3):

$$u = \frac{MV \pm mv}{M + m}$$

This result naturally follows from the conservation of momentum (the quantity of motion) of two bodies before and after the impact[7]:

$$MV \pm mv = u(M + m)$$

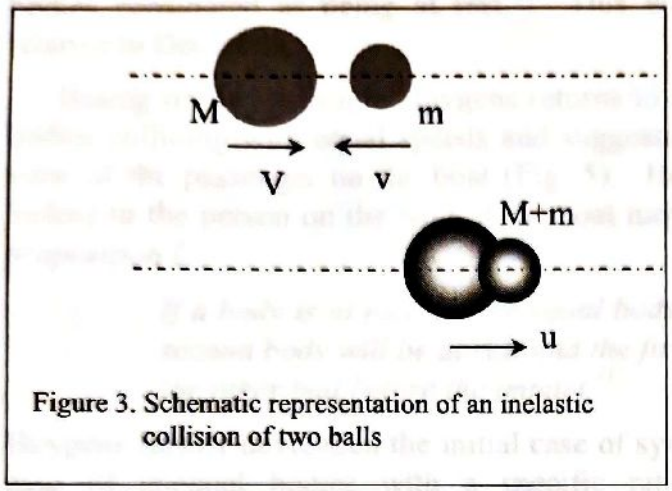

Figure 3. Schematic representation of an inelastic collision of two balls

As mentioned, sign ± adopted to signify the two cases of shared and opposite directions of movements of the bodies prior the impact. This was a significant improvement of Descartes' results, which separated motion as quantity (speed) from its direction (determination). For Wallis the quantity of motion became *algebraic quantity* – one step before vector quantity, as we use today.

The second step: elastic collision


Christopher Wren

Wallis addressed also the elastic collisions that took place when the hard or elastic bodies collide. He, however, described precisely only a special case of such collision: when two equal bodies move in the straight line in opposite directions (Fig. 4). In such a case, each body rebounds and the bodies exchange velocities. The further progress in the empirical account for the elastic collision was due to Christopher Wren (1632-1723)[8] in 1669. The experiments were replicated and extended by Mariotte in France in 1677.

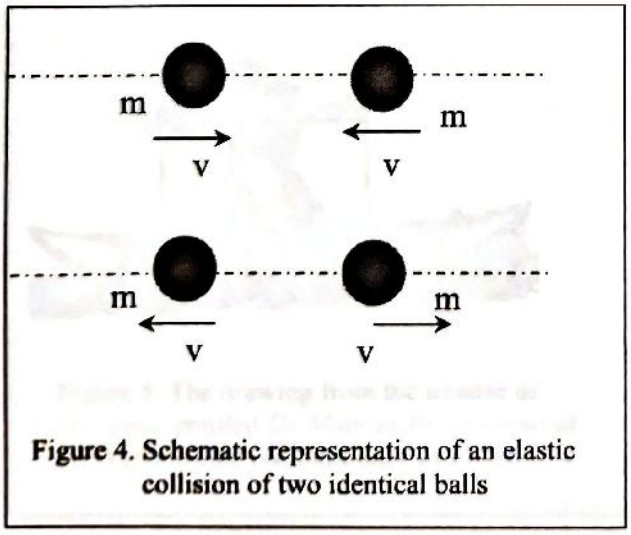

Figure 4. Schematic representation of an elastic collision of two identical balls

However, the most mature account for the elastic collisions was due to Christian Huygens[9] who was able to apply the newly introduced by Galileo

[6] In fact, Wallis formulated the rule in terms of weights (P_1 and P_2). Only Newton, several years later, distinguished between mass and weights as different concepts.

[7] For the quotations from the treatise of Wallis, look to Dugas (1988, pp. 172-175).

[8] Christopher Wren was especially celebrated as the architect of London of that period, but being a cultural person with broad intellectual interests, he contributed also to physics.

[9] Christian Huygens – an outstanding physicist, one of the founders of the modern science and classical mechanics in the 17th century.

principle of relativity to demonstrate the general rule of the elastic collision.

Christian Huygens

For this, he started from the known to all obvious case: two equal hard bodies approach each other at equal speeds. The result – their receding at the same speeds (in other words, exchanging velocities – the first rule of Descartes) – was postulated (Huygens called it *hypothesis*). Then Huygens used another postulate stating that all motions and the claim regarding their equality or inequality should be considered only relatively, that is, with respect to other bodies considered as being at rest.[10] This was a breakthrough relative to Descartes.

Basing on this principle Huygens returns to the postulated collision of two equal bodies colliding with equal speeds and suggests to imagine that this happens in the view of the passenger on the boat (Fig. 5). He also asked how the same event is looked to the person on the bank if the boat moves at the same speed v. He inferred *proposition I*:

> *If a body is at rest and an equal body collides with it, after the impact the second body will be at rest and the first will have acquired the velocity that the other had before the impact.*[11]

Huygens further developed the initial case of symmetrical collision to a more general case of unequal bodies with a specific ratios between velocities and masses (*proposition VIII*):

> *If two bodies moving in opposite directions inversely proportional to their magnitudes collide with each other, each one rebounds with the velocity that it had before the impact.*[12]

This more general result theoretically refuted Descartes' rule 6, matching the well known empirical fact. It is easy to see that this claim matches the principle of momentum conservation.

Figure 5. The drawing from the treatise of Huygens, entitled *De Motu ex Percussione* of 1656 (published posthumously in 1703).

Indeed, if M and m are masses of the bodies moving with the velocities u and v that collide, and the masses ratio is $\frac{M}{m} = -\frac{v}{u}$, then the general claim of momentum conservation

$$Mu + mv = Mu_1 + mv_1$$

is satisfied exactly by the solution for the velocities u_1 and v_1 after collision as follows:

$u_1 = -u$ and $v_1 = -v$ (Fig. 6), as can be checked by direct substitution.

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[10] It might look for us as a trivial statement. However, it is this unconditional relativity of motion that discharges the medieval concept of *impetus* often understood as a sort of absolute feature of a moving body – a "charge" of motion.

[11] Dugas, R. (1955). A History of Mechanics. Dover, New York, p. 177. Quotations from Huygens' *De Motu corporum ex percussione* (1700).

[12] Ibid, p. 178.

Following Wren who observed realization of this rule in experiment, Huygens termed such velocities – *proper velocities*.[13]

Proposition 11 was, however, of special interest. We read there:[14]

"In the mutual impact of two bodies the sum of the products of the masses into the squares of the respective velocities is the same before and after impact."

If we rewrite this claim in the modern symbolic form we get:

$$Mv^2 + mu^2 = Mv_1^{\ 2} + mu_1^{\ 2}$$

In fact, this was the claim of conservation of the quantity defined by Leibniz (1669) that he termed *vis viva* (the living force) in the collision of hard (elastic) bodies. This was an important step in the long way of physics towards the principle of energy conservation. We can get an idea of how this conservation was obtained.

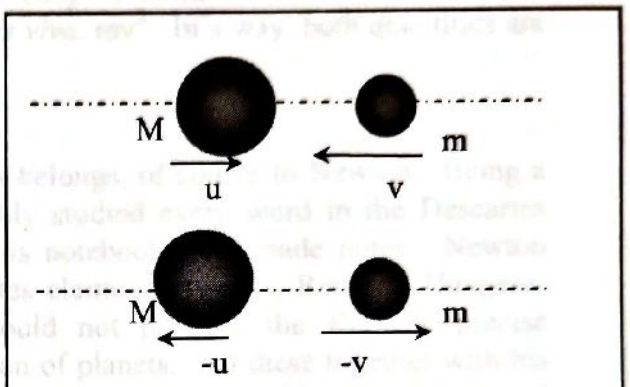

Figure 6. Schematic representation of an elastic central collision between two unequal balls

As already established by Wren empirically, the collision of two bodies could be characterized by the change of their relative velocities in the impact, regardless their masses. If the initial velocities of the bodies were, say, u and v, and the terminal velocities were, say, u_1 and v_1, then we should consider the ratio:

$$e = \frac{v_1 - u_1}{u - v}$$

Clearly, then, when the bodies collide softly, $v_1 = u_1$ and the ratio nullifies (e=0). The experiment showed that e could be at most approach e=1. This is the case of *elastic collision*. Therefore, the rule obtained for the elastic collision of two bodies was:

In an elastic collision, the relative velocity of the two colliding bodies reverses.

For all intermediate cases of collision, the values of e (justifiably termed as the *coefficient of restitution*) fall into the interval between zero and one:

$$0 \le e \le 1$$

Consider the elastic collision, then:

$$v_1 - u_1 = u - v \quad \text{or} \quad v + v_1 = u + u_1$$

At the same time, the conservation of momentum implies:

$$Mu + mv = Mu_1 + mv_1 \text{ or } M(u_1 - u) = -m(v_1 - v)$$

Multiplication of the last two equations yields:

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[13] For the way in which Huygens himself demonstrated this proposition see in Dugas (op.cit., p. 179).

[14] Wolf (1968). *A History of Science, Technology and Philosophy in the 16th and 17th Centuries*, Peter Smith, Gloucester, Mass., p. 233.

$$M(u_1^2 - u^2) = -m(v_1^2 - v^2)$$

Or, rearranging the terms provides:

$$Mu^2 + mv^2 = Mu_1^2 + mv_1^2$$

So, Huygens was able to arrive to the rule of collision, which in fact was the claim of conservation of the kinetic energy in the elastic collision (the exact statement should, of course, include coefficient 1/2 in all terms, which does not change the equation quantitatively).

The importance of this result was beyond the claim of conservation, important by itself. This result put the end to the debate between Descartes (Newton) and Leibniz regarding what the quantity that should be adopted as the "true" characteristic of motion: the quantity of motion: mv, or the *vis viva*, mv^2. In a way, both quantities are required: energy and momentum.

Final refutation – Newton

The final word in the debate with Descartes belongs, of course to Newton. Being a teenager student of Cambridge he thoroughly studied every word in the Descartes Principles of Philosophy, copied them to his notebooks and made notes. Newton knew of course about refutation of Descartes claims by Wallis, Ren and Huygens. The qualitative approach of Descartes could not produce the Kepler's precise *mathematical* statements regarding the motion of planets. All these together with his own views and the debates with Hooke in the Royal Society of London stimulated Newton and brought him to create the fundamental treatise of human culture – *The Mathematical Principles of Natural Philosophy* – Newton's answer to the *Principles of Philosophy* of Descartes. Newton started his debate with Descartes already from the title: ***Mathematical*** *Principles* instead of *Principles*, and ***Natural*** *Philosophy* instead of *Philosophy*.

Right from the beginning Newton presents his laws of Nature which had to replace Descartes' ones. Here are they both in comparison:

Laws of nature in Descartes' Principles	Laws of nature in Newton's Principia[15]
The first law of nature: that any object, in and of itself, always perseveres in the same state; and thus what is moved once always continues to be moved. *The second law of nature: that every motion of itself is rectilinear; and hence what is moved circularly tends always to recede from the center of the circle it describes.* *Third law: that a body, in colliding with another larger one, loses nothing of its motion; but, in colliding with a smaller one, loses as much as it transfers to that one.*	*Law I. Every body perseveres in its state of being at rest or of moving uniformly straight forward except insofar as it is compelled to change its state by forces impressed.* *Law II. A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.* *Law III. To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always opposite in direction.*

[15] Newton, *op.cit.*

What were the major changes introduced by Newton in the laws? The changes were numerous and essential.

1. The first laws might seem rather similar. However, a closer look reveals essential differences.[16] Newton' laws includes relation of the body to the impressed force and describe the tendency of the body to preserve the state of motion or rest. These states are claimed to be equivalent unlike the perception of Descartes. This was the paradigmatic shift in physics knowledge which became among the central features of modern physics.
2. The second and third laws of Descartes were removed Newton. In a way, they became direct implications of the second Newton's law.

   Indeed, the second law of Descartes claims that the body "moved circularly tends to recede from the center of the circle it describes". This was rather a debt Descartes paid to the old special status of circular motion considered since Greek Philosophers being superior to any other motion. This status was reconsidered in Newton's mechanics and equated to any other curvilinear motion (such elliptical, parabolic, or hyperbolic trajectories). As to the "tendencies" they were removed in favor of using forces.

   Regarding the third law, stating the conservation of (quantity of) motion, it was essentially changed. The principle of conservation of quantity of motion (this time a quantity sensitive to direction of motion – *vectorial*, in our terms) was stated in Corollary III:[17]

   > *The quantity of motion, which is determined by adding the motions made in one direction and subtracting the motions made in the opposite direction, is not changed by the action of bodies on other.*

   This claim is a direct implication of the second Newton's law for a closed system of bodies.
3. The fundamental fallacy of Descartes who considered the interaction of bodies as asymmetrical process (in terms of winners and losers) was removed Newton in his third law. Perhaps within the debate with Descartes Newton stated this law in a separate claim although unlike previous laws, which were stated as axioms (that is without demonstration), the third law was proved by Newton basing on the first one.[18] Interactions between any two bodies were stated to be symmetrical: action is equal to reaction.

Unlike Descartes' laws, Newton's laws matched the experience quantitatively and therefore were unanimously preferred by scholars to Descartes' ones. All the empirical results of Wallis and Ren regarding collisions matched to Newtonian mechanics with the high level of accuracy.

As to the results of Huygens regarding the conservation of vis viva (kinetic energy) it took more time. Although not introduced by Newton, this concept perfectly matched to his theory and was included later to the body of classical mechanics.

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[16] Galili, I. & Tseitlin, M. (2003). Newton's first law: text, translations, interpretations, and physics education. *Science and Education*, 12 (1), 45-73.

[17] Newton, *op.cit.* p.420.

[18] Newton, *op.cit.* pp. 427-428.

Questions for reflection:

1. Present the rationale of Descartes by which he treated collisions of hard bodies.
2. Criticize the rules of collisions.
3. Justify the name for the coefficient $e = \frac{v_1 - u_1}{u - v}$ as *coefficient of restitution.*
4. Exemplify the violation of principle of relativity by Descartes' rules of collisions.
5. List and discuss the differences between Descartes' and Newton's laws of motion.

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Historical and philosophical perspective

Descartes' knowledge of mechanics – Ontology

Physics includes several fundamental theories, not too many, but they very powerful, inclusive and useful. Each of them creates a unique picture of the world. To understand the essence of such pictures, their features and scope, one should peep into the history of their creation and conceptual consolidation. In doing this we, the consumers from the future, obtain an advantage of looking backwards, because today we know much more than the pioneers knew and therefore can easier understand the meaning of things that they introduced with relation to the subject matter, expanded and developed.

Classical Mechanics creates one of such world pictures. In it, all bodies influence each other causing their changes and movements. The fundamental questions mechanics asks are what laws and principles govern these movements and changes of objects, and how exactly different bodies influence each other. Scholars launched two major ways of thought in this regard: *action at a distance* (when objects were ascribed an ability to influence each other through space without asking – how?) and *action at a contact* (when things influence each other by actual impact – touch or collision).

Here we have considered the major historical fragments of the physical theory of collisions (repercussions, as it was sometimes termed) which was a representative model of interactions.[19] The progress actually started in the 17th century in Europe,

[19] Wesfall, R.S. (1989). *The Construction of Modern Science.* Cambridge University Press, Cambridge.

when the first rules of collisions were established. On our days, students use these rules when they learn mechanics and solve various problems about collisions. They consider them as a practical situation to simply apply the known mechanics laws. However, the genuine understanding of physics requires more than comprising and solving equations but rather recognition of the main physical underpinnings.

Two major ontological principles made by Descartes were emphasized in our excurse: (1) the *rectilinear uniform motion* and the *state of rest* were announced to be fundamental states preserved as long as the external agent causes their change and (2) the principle of *conservation of quantity of motion* expressed in collisions between bodies. These were the new laws of motions preceded to Newtonian ones.

Among the central claims of the modern science of motion is the claim of equivalence of the rectilinear uniform motion and the state of rest. This statement is very much non-obvious to commonsense understanding: how could be motion and rest equivalent? In Descartes writings one observes that this understanding was not reached. It is therefore not surprising that a mere claim of rest-motion equivalence made by a teacher normally does not provide understanding of students. The change may come if one recapitulates the construction of this idea, the first steps made by the bright minds of the past within the scientific discourse of scholars, through their tries and original accounts of motion.

Furthermore, the concept *quantity of motion* was understood differently to the modern concept. Unlike momentum, a product of mass and velocity, a vector quantity, the quantity of motion was defined as a product of mass and speed. Direction of motion was separated to independent concept.

Among other discrepancies of Descartes' understanding of interaction is the non-symmetrical nature of interaction between bodies. One body should overcome the other by a "greater force" and impose the change of motion on the one that lost. The principles used to determine the victory were also specific. In the case that one body is at rest (absolute fact) the winner from the two colliding bodies is the one with greater mass. In the case of two moving bodies one should compare the quantities of motion in the bodies to choose the winner. All this conception was removed by Newton's conception of total symmetry between interactive bodies in terms of the forces they apply each on the other: action force was stated to be exactly equal to the reaction force.

Furthermore, the *rest-motion equivalence* is inherently related to another not less important idea – *the relativity principle,* one of the central principles of the modern and classical physics. The correct conceptual account of collisions is in fact related to both principles. When this relationship was essentially used in the account of collision the mechanics of Descartes collapsed. The conceptual change took place exactly in the transfer from Descartes to Huygens. It was the latter who reduced various cases of collisions between identical bodies to a single case – a symmetric collision.

In his second law Descartes addressed the circular motion as if it has a special status. This was a reflection of the very old tradition in the Natural Philosophy to consider circular motion as a natural and superior one (the "noblest", "eternal" etc.). This was still the view of Galileo. Descartes was the first who pointed to the tendency "*always to recede from the center of the circle*" and proceed th emotion along the straight line. The circular motion ceased to be "natural" but accompanied

with tension in the constraint (the rope). From then it waited for Newton who introduced centripetal force as the cause of circular motion. It was Newton who totally removed the superior status of circular motion which remained only a special case (albeit the simplest one) of the curvilinear motion.

The account of post-Cartesian researchers addressed collisions of bodies in wider span of different cases: when the two colliding masses stuck together and when they separated following the impact with different velocities. These scientists came to understanding that during the mechanical process of collision the quantities that conserve during were: the quantity of motion (momentum mv – a vector) and the kinetic energy (vis viva mv^2 – a scalar). In the 17th century scholars did not know the relationship between the two. The momentum was conserved in any collision and the kinetic energy – only in elastic ones.

In the excurse we addressed the work of the pioneers from the 17th century: Rene Descartes, Wallis, Wren, Huygens, and Marriott who paved the way to Newton and the contemporary knowledge in the realm of mechanics. Therefore, they can help us to better understand the conceptual foundation of the subject of basic laws of mechanics in its critical features.

Questions for reflection:

1. Discuss the points in which Descartes' knowledge of mechanics was different from classical mechanics.
2. Demonstrate the violation of rest-motion equivalence by Descartes (in the rules of collisions) by violation of the principle of relativity.
3. Show that rest-motion equivalence and the principle of equivalence are incorporated in the Newtonian framework of mechanics.

Descartes' way to knowledge – epistemology

Descartes[20] was a unique person of bright mind who combined being a philosopher and physicist. His way in establishing the laws of motion reflected his worldview in which philosophical principles are deeply interwoven with physical statements, and they influence each other. However, it is the philosophical principles that constitute foundation of knowledge. The concrete physical laws, believed Descartes, can be deduced from the basic principles.

[20] Rene Descartes (1596-1650) – French philosopher, mathematician, physicist and physiologist, the founder of modern philosophy of rationalism.

This approach to knowledge construction is defined in the philosophy of science as *rationalism*. Plato, the great Greek philosopher of the 4th c. BC, is considered to be the founder of such approach to the scientific knowledge. Descartes constructed knowledge of mechanics using this approach and in he tried to be consistent with it also on the expense of correspondence the empirical evidence. Euclidean geometry may serve a model of such approach to knowledge construction.

In this approach Descartes apparently took the side of Plato and thus entered in a bitter argument with the opposite philosophical approach in the philosophy of science – *empiricism*, which stated the prerogative in knowledge construction to the empirical evidence and inductive inferences. This trend is traditionally identified with Aristotle, another great Greek philosopher of the 4th c. BC.

In his *Principles of Philosophy,* published in 1644, Descartes developed his version of the laws of motion and deduced from them the rules that should govern collisions.

In some of the stated principles Descartes was right: the uniform rectilinear motion is indeed a state of motion, which does not need any cause to be preserved. Furthermore, Descartes was right in his conjecture that the interaction between the bodies cannot change the quantity motion – the principle of motion conservation. However, in other aspects Descartes was wrong.

He failed to equate between the rectilinear uniform motion and rest, and he did not adopt the principle of relativity established by Galileo. Descartes failed to understand that quantity of motion includes direction (and not independent of it). He failed to grasp the symmetry of action-reaction. For him one body won the other in a collision and imposes on it the change of motion. These misunderstanding made almost all of his predictions wrong, although they presented a certain progress from the Aristotle's theory of mechanics as well as from the medieval theory of impetus practiced by scholars for about two thousand years.

What was worse that being blindly devoted to rationalism, Descartes could not correct his theory basing on the empirical evidence, even in cases when the discrepancy was just striking. Without any attempt to evaluate the masking influence of the medium, he blamed the absence of vacuum in nature in the fact that his laws do not hold in many cases. This explanation cannot unlimitedly "save the phenomena" in science. In his letter to the French translator Descartes explained his view on the knowledge construction in science and philosophy – rationalism:[21]

> *I should have here shortly explained wherein all the science we now possess consists, and what are the degrees of wisdom at which we have arrived. The first degree contains only notions so clear of themselves that*

[21] Descartes, R. (1644/1983), op.cit. Letter from the Author to the Translator of this Book, pp. XVII-XVIII.

> *they can be acquired without meditation; the second comprehends all that the experience of the senses dictates; the third, that which the conversation of other men teaches us; to which may be added as the fourth, the reading, not of all books, but especially of such as have been written by persons capable of conveying proper instruction, for it is a species of conversation we hold with their authors. And it seems to me that all the wisdom we in ordinary possess is acquired only in these four ways; for I do not class divine revelation among them, because it does not conduct us by degrees, but elevates us at once to an infallible faith. There have been, indeed, in all ages great minds who endeavored to find a fifth road to wisdom, incomparably more sure and elevated than the other four. The path they essayed was the search of first causes and true principles, from which might be deduced the reasons of all that can be known by man; and it is to them the appellation of philosophers has been more especially accorded.*
> ...

Here, after naming the four common ways to knowledge: (1) the obvious known, (2) experience of senses, (3, 4) information from other people, oral or written. To all these Descartes preferred the fifth way that he saw as his major way of investigation: revealing the principle from which all the rest could be explained by deduction. Regarding such principles Descartes proceeded:[22]

> *Two considerations alone are sufficient to establish this--the first of which is, that these principles are very clear, and the second, that we can deduce all other truths from them; for it is only these two conditions that are required in true principles.*

Descartes exemplifies and summarizes, stating that his approach is sufficient to gain any true knowledge:

> *...Those are all the principles of which I avail myself touching immaterial or metaphysical objects, from which I most clearly deduce these other principles of physical or corporeal things, namely, that there are bodies extended in length, breadth, and depth, which are of diverse figures and are moved in a variety of ways. ...we can deduce from them [principles] the knowledge of whatever else is in the world. ...I think I have so explained all of which I had occasion to treat, that they who read it attentively will have ground for the persuasion that it is unnecessary to seek for any other principles than those I have given, in order to arrive at the most exalted knowledge of which the mind of man is capable.*[23]

As one may see, Descartes did not save words (and did not exaggerate in modesty) to convince that his way in science was the correct one. Seemingly, however, one may prefer to this extended description, that happened to be limited in truth as being a sole bases for construction new knowledge in physics, a few words Descartes wrote in the end of presenting the seventh rule of collisions:[24]

> *And the demonstrations are so certain that, even if experience seemed to show us the contrary, we would nevertheless be obliged to place more faith in our reason than in our senses.*

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[22] *Ibid.*
[23] *Ibid.*
[24] Descartes (1644/1983), *op.cit.*, p. 69.

Target group, curricular relevance and didactical benefit

The developed unit addresses first of all physics teachers, pre- and in-service. This is because the regular curriculum does not include knowledge which is considered today obsolete and incorrect. Indeed, the presented laws of motion were replaced by those of Newton – the nucleus of physics curriculum of mechanics. What is, then, valuable for physics teaching in Descartes' theory of motion wrong and replaced? Here we summarize some aspects of curricular relevance and educational benefit of this excurse.

- Descartes' *Principles of Philosophy* belongs to the important texts of the Western culture in philosophy and science. The chosen fragments are relatively easy for understanding, and through mediation by a teacher, students may touch on the fundamentals of science, both ontological and epistemological in nature, consolidated during the scientific revolution of the 17th century. This presents introduction into the culture of science.

- Within the perspective of *internal* culture of physics[25] the theory Descartes belongs to the periphery of the Newtonian classical mechanics and as such it by contrast strengthen the major ideas of Newtonian paradigm of mechanics and the same virtue it supports the meaningful learning of classical mechanics.

- The considered text presents seeking understanding of Nature for its own sake, regardless pragmatic advantages. This approach demonstrates the true spirit of science, and provides a contrast to the often prevailing consuming orientation.

- Descartes represents the rationalist epistemology in physics knowledge. This approach is not the one adopted by science and by its critique one can emphasize the lacking part of the modern *scientific method* which combines rational and empirical approaches.

- This excurse illustrates how and why physicists rejected Cartesian rules of collision and corrected his principle of momentum conservation. The critique was made empirically by Wallis and Wren and theoretically – by Huygens. Learners will have a chance to experience and learn both types of examination. While considering the necessity of experiments, the type of experiment with controlled variables can be emphasized, in opposition to simple observation.

- Despite its erroneous aspects, the Cartesian theory presents the progress made in overcoming the medieval understanding of motion and referring to the uniform rectilinear motion as a state of matter (not a process). The latter does not need a supporting cause (an external mover or internal impetus) as was practiced by scholars before the 17th century. This was the paradigmatic change in physics.

- Among the conceptual mistakes of Descartes was his ignoring the vector nature of quantity of motion (the product of mass and speed, instead of – mass and velocity). Revealing this mistake in concrete examples emphasizes the correct knowledge.

- Descartes' approach essentially draws on the *conservation of motion*. Although regarding incorrect concept, the approach of using conservation principles was new

[25] Tseitlin, M. & Galili I. (2005). Teaching physics in looking for its self: from a physics-discipline to a physics-culture, *Science & Education*, 14 (3-5), 235-261.

and remained as most fundamental way to treat motion and solve problems. Huygens' discovery of the conservation of *vis viva* in the elastic collisions (practically conservation of the kinetic energy) further benefit the students account for collisions.

- Descartes erroneously used the mechanism of "competition" between the bodies in which one should apply a greater force to cause the change of motion to the other. This presents a common misconception among the students and thus could be treated through cognitive resonance caused by critique of Descartes.

- By contrasting with the classical mechanics, Cartesian theory of collisions may cause better understanding. Instead of solution of standard equations, our excurse calls for thinking about the difference between the *momentum*, always conserved in collisions and the *kinetic energy* – conserved only for the elastic collision. Students learn in a sufficiently simple context to distinguish between the two types.

- The theoretical critique of Descartes was performed by Huygens who essentially used the relativity principle of Galileo – the central principle in the modern physics. In this Huygens was ahead of his time, including Newton. In physics class today, practically no problem requires application of the relativity principle. The method applied by Huygens could be beneficial in providing a rare opportunity to make use of this principle.

- The materials of this historical case demonstrate the need of modesty in scientific claims: despite of the great intellectual power many of Descartes' principles appeared to be wrong. Discussing these materials may be educative to young learners, thus performing their enculturation into physics.

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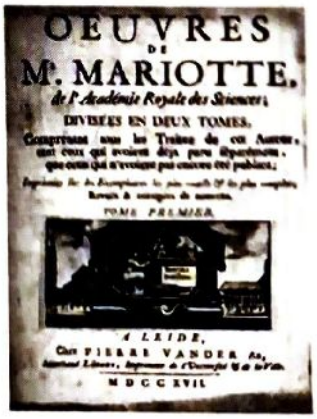
Activities, methods and media for learning

The major mode of presenting this excurse could be a series of interactive lectures incorporating discussions. It is recommended to precede the discussions with a questionnaire asking students for their account for what may happen in collisions of various types (light and heavy, hard and soft bodies).

It is of special importance to elaborate on the role of principles of conservation in accounting for a complex and/or unknown interactions, as it took place in the course of history (conservation principles were practiced much before the nature of interaction was revealed). This topic provides an opportunity of addressing ontological as well as epistemological aspects of physics knowledge. The questions to ask could be taken from the published researches (including quoted below) that investigated students' knowledge. For example, one may ask regarding the conditions of applicability of the conservation of kinetic energy in comparison with those for the conservation of momentum.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This excurse unfolds the history, which preceded the establishing of the classical mechanics by Newton. When mastered by the practicing teachers this knowledge will provide them with the background illuminating the material they directly teach at classes. The text of the case could be distributed among the group of teachers or students as a home assignment. The preliminary reading can be

followed by a classroom discussion, during which the teacher should mediate the major points of importance as formulated in the previous section.

Various fragments from this excurse could be given as separated topics in subject matter for students report and discussion. Such are *"Descartes' laws of motion", "Descartes' laws of collision", 'The empirical laws of impact by Mariotte, Wren, and Wallis", "Huygens' treatment of collisions and Galileo's principle of relativity".*

Epistemological aspects of the nature of science could be addressed in reports that choose the topics: *"Rational versus empirical knowledge with regard to collisions of bodies", "The role of mathematics in the scientific revolution of the 17th century".* Students may discuss the topic when they accept roles representing Descartes or his opponents either empirical (Wallis, Wren, Mariotte) or theoretical (Huygens). This presentation should open a discussion in which personal conceptions of students are revealed and elaborated. This polemic may mimic the scientific debate as arranged by the teacher.

Edme Mariotte

Possible empirical activities may include the exploration of impact between two different bodies using the apparatus of Mariotte (two pendulums in a contact). Another activity could be with using the ballistic pendulum constructed by students as suggested by Mariotte and realized by Robins in the 17th century (Fig. 7). Students may discuss the special need for ballistic pendulum and explain its functioning.

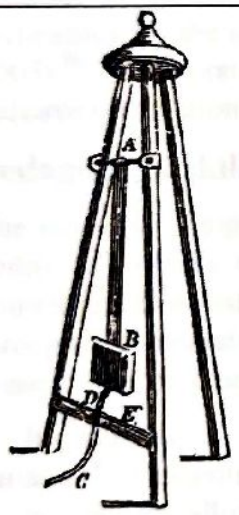

Figure 7. Ballistic pendulum by Robins.

Another story might be interesting to mention. Among the first researchers tried to study collisions was the Czech scientist Jan Marci from Prague prior to Wallis and Wren in London in 1668. A cannon fired a ball that stroke another ball placed on a stone table. After the impact the first ball stopped and the other started to move ahead in the direction of the original shot. Although this demonstration was not anything else but repeating the collision of two billiard balls, the context of a cannon ball that completely stopped only by an equal shot at rest, greatly impressed the observers being anti-intuitive.

The popular apparatus demonstrating collision of hard objects is known under the name of Newton's cradle (Fig. 8). It is comprised of several identical pendulums suspended in a line, touching each other. In fact, this device is an extension of Mariotte apparatus to measure velocities of two colliding balls (Fig. 1), but possessing wider opportunities of experimenting.

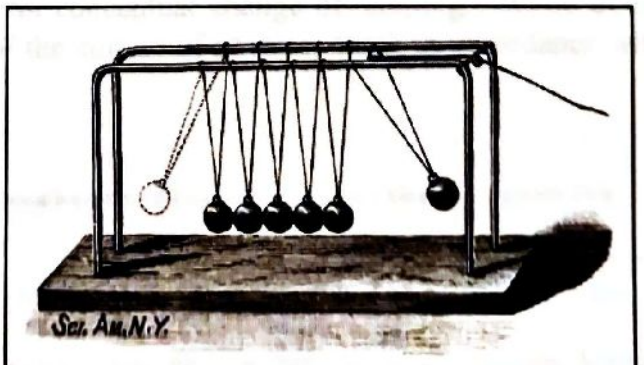

Figure 8. Newton's cradle

After a series of trials, students discover the rule: the same number of balls that were initially raised up from one side of the line, and

stopped by the rest of the balls rise at the opposite end and reach almost the same height. This result is of course in accordance with the principle of conservation of momentum in the collision.

The whole phenomenon seems to be similar to the ball falling from a height and colliding with another ball at rest: the initial ball stops and the other proceeds with the velocity of the first ball. In the case of several balls in the middle, which remain at rest, one might think that these balls in the middle do not participate at all and merely transfer the momentum. To discover the true scenario of this phenomenon one might tie together the balls in the middle, which seem not participating in the collision. For example, if the line has six balls (as in Fig. 8) and we raise one ball at the left, a single ball rise up at the right end of the line, when the first ball stops. If now we tie together the four balls in the middle of the line, the result of the impact of the first ball with the line will be very different – all balls start to swing after the impact. Careful observation and additional thought may reveal that in the case of free balls (not fasten together) a series of collisions takes place, each time between two adjacent balls. The collision travels along the line until the ball at the right end rise up. The succession of collisions is fast enough to be missed without knowing what to look for.

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Obstacles to teaching and learning

The presented excurse to the history of mechanics goes to the 17th century original treatises of Descartes and Huygens. Using such historical materials may cause difficulties for the obsolete language, style, and concepts used (e.g. Galili & Tseitlin 2003).[26] It is upon the teacher to mediate these texts to the students. This activity requires preparation of teachers in a special workshop.

Pedagogical skills

The major pedagogical skill required from the teacher to present this excurse is an ability to mediate the knowledge, meaning to encourage students' construction of knowledge in a dialogue converging to understanding of the goal scientific concept through a comparative analysis of the alternative theory (Descartes) preceded to the presently adopted one (Newton).

In educational context this means that mastering the skill of teaching by variation[27] is essential. Teacher should be sensitive to the ideas his/her students' hold on the subject allowing their discussion. By this the teacher creates a space of learning incorporating various alternative conceptions. There, students are stimulated to discern the right ideas and perform conceptual change of learning. While doing that the teacher serves as an agent of the culture of physics, much in accordance with the ideas of Lev Vigotsky.[28]

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[26] Galili, I. & Tseitlin, M. (2003). 'Newton's first law: text, translations, interpretations, and physics education'. *Science and Education*, 12 (1), 45-73.

[27] e.g. Schur, Y. & Galili, I. (2009). Thinking Journey – a New Mode of Teaching Science. *International Journal of Science and Mathematics Education*, 7(3), 627-646. Also see Marton, A. & Tsui (2004). *Classroom discourse and the space of learning*. Mahwah: Lawrence Erlbaum

[28] Vigotsky, L. (1934/1986). *Thought and Language*. The MIT Press, Cambridge, Mass.

Research evidence

Students' knowledge of the laws of motion and the account for collision by conservation laws was investigated in many researches. For example, Galili & Bar (1992) reported that students associate the uniform rectilinear motion of objects with acting "moving" force. The required conceptual change could be encouraged in the discussion on Descartes' and Newton' first laws of motion.

In the studies by Grimellini-Tomasini, Pecori-Balandi, Pacca, & Villani (1993) from the University of Bologna and the University of San Paolo, as well as the research by Sasson (2006) from the Hebrew University of Jerusalem, the researchers detected and reported that many students of high school physics classes have difficulties in applying conservation laws to account for collisions.

In particular, there is a frequent confusion between the conditions that allow using the principle of momentum conservation and those allowing application of kinetic energy conservation.

Teaching with using relevant historical materials may improve students' views on the nature of science, in particular, their image of the scientific method as combining rational and empirical approaches in physics exploration. The research evidence of such an impact was reported in Galili & Hazan (2001).

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